

A
COMPENDIOUS COURSE
OF
PRACTICAL MATHEMATICS.

Particularly Adapted to the
USE OF THE GENTLEMEN
OF THE
ARMY AND NAVY.

IN TWO VOLUMES.

For the most part translated from the *French* of P. Hoste
Professor of Mathematics in the
Royal Academy of Toulon,

By the late *WILLIAM WEBSTER.*

The THIRD EDITION,

Corrected and improved by S. CLARKE, Teacher of
Mathematics, and Author of *An easy Introduction to
the Theory and Practice of Mechanics.*

VOL. I. Containing,

I. Elements of EUCLID.	IV. MECHANICS.
III. TRIGONOMETRY.	V. FORTIFICATION.
II. Practical GEOMETRY.	VI. GUNNERY.

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B. LAW, and W. NICOL. MDCCLXIX.



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TO HIS GRACE

J O H N,

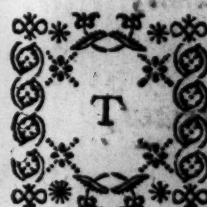
Duke of ARGYLE and

GREENWICH,

Master-General of

HIS MAJESTY'S Ordnance, &c.

May it please Your GRACE,

 THE collection of Tracts which form this Volume, containing such parts of Mathematical knowledge as are properly the study of a Soldier, seems to claim a kind of privilege, to Your Grace's protection, as a General, and Master of the Ordnance : Nor can there be a greater inducement to the Gentlemen of the Army to regard these necessary Qualifications, than the Countenance of Your Grace, whose great Example they will be justly ambitious to follow.

A R M S have been Your early exercise, and A R T S Your constant study : So that the eminent Figure Your Grace has

DEDICATION.

has long made, both in the Field and Senate, is a topick upon which even a cold Imagination might with ease expatiate : But tho' just Praise is undoubtedly a grateful Incense, yet there is a certain delicacy in the Offering, which I dare not attempt.

I therefore humbly beg pardon for the presumption of this Address, and crave leave, with the most profound respect, to subscribe myself,

May it please Your GRACE,

Your Grace's most Obedient,

And most Humble Servant,

William Webster.



T H E

A U T H O R ' S P R E F A C E .

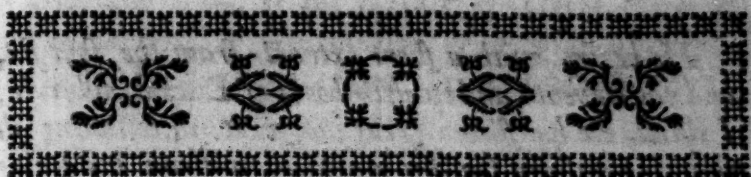
B E I N G obliged by my Station to study the properest means of instructing the Marine Officers in such parts of the Mathematics as are necessary for their Qualification, I thought the most effectual method would be to lay before them a Collection, wherein they might both see what they had to learn, without troubling themselves with any other Books, and might with ease revise what they had already been instructed in.

I considered, that this Collection ought to be as short as possible ; that the Persons for whom it is intended might not be discouraged with the length of the work ; and I may reasonably hope that every Gentleman concerned will endeavour to render himself duly qualified, when he observes, that the whole course of his Mathematical Studies are comprehended in two little Volumes.

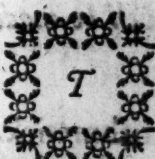
In the entrance upon each Subject I have let nothing pass without clear explication and demonstration ; but in the progress, I have chose to leave the Reader the pleasure of discovering by himself some of the most easy conclusions ; of which, however, I have given him proper intimations.

I have not been scrupulously exact in rendering my method of demonstration perfectly uniform, and in deducing my proofs immediately from first principles, or the next preceding Propositions; but have rather made it my chief endeavour to avoid every thing that might embarrass, or discourage my Readers: And when several demonstrations have offered themselves in proof of the same Proposition, I have always chose that which appeared to me most intelligible and short, rather than that which perhaps might be thought more exact.





T H E P R E F A C E.


T

 HIS short Course of practical Mathematics, which I have ventured to translate from the French, for the benefit of the English Reader, will, I hope, by its brevity and perspicuity, recommend itself to such Gentlemen as think some knowledge in these useful Sciences a laudable Accomplishment. How necessary this Study is, especially in the Articles which make up this Collection, to those who would rightly qualify themselves either for the Army or Navy, is too evident to be insisted on; but how conducive the present Performance may be to that useful purpose, is not for me to determine; as a Translator, I am become a party in the Design. I shall therefore presume no farther, than humbly to offer a word or two in behalf of my Author, with respect to the several parts of his Work, and to subjoin, as I go

on, a short Account of the liberties I have taken in rendering it into English.

He begins with a short extract fram the first six, and the eleventh and twelfth books of Euclid, wherein, I think, he has shewn great judgment, in collecting all that is necessary for his purpose, without burdening his Reader with any propositions foreign to his design; so that I think it may be justly said, there is nothing extant of the like kind, at once so compleat and so concise: Nor do I judge it unworthy of remark, that the propositions stand numbered as they follow in the books of Euclid. With regard to the translation of this part, I have one thing to advertise the Reader, viz. That whereas the Author had referr'd his extract from the fourth book of Euclid, to his tract upon Practical Geometry, I thought it better to give all the Elements together, since references might as well be made to that Part as the rest.

The second subject of our Author is Arithmetick, upon which having myself wrote more fully, I entirely omitted his Tract; yet I hope it is not ill supply'd with my Arithmetick in Epitome, which may serve as the first Volume of this Course.

Trigonometry is his third tract, in which he has pretty copiously handled the Doctrine of Triangles, both plain and spherical, and has therein also given a short, but clear, explication of the construction and use of Logarithms.

Practical

Practical Geometry is his next subject, wherein he succinctly treats of the manner of taking Heights and Distances, of the method of making Surveys, and of the measurement of Surfaces and Solids. This part in the Original contains likewise some extracts from the fourth Book of Euclid, which, as I before observed, in the Translation, I have ranged in their proper order among the Elements, with some additions. He has likewise therein given the description of some instruments, for which I must refer the Reader to the Appendix I have added at the end of the last Volume.

The Use of the Globes is the next tract in the order of our Author; but being desirous to comprise all that more immediately regarded the Land-Service in one Volume, I have placed this Tract in the beginning of the last Volume, as more adapted to Maritime Affairs. All I shall say of this Treatise is, that it is very short, and yet I think sufficiently intelligible.

Next follows a tract upon Mechanicks, the shortest and most intelligible I have ever met with. There is one Article in this Treatise I have taken the liberty to alter (tho' indeed I rather believe it to be an error of the Printer, than a mistake of the Author) since the change of but one word, viz. autant into moins, that is as much into less, makes the observation just, and the sense consistent with the Corollary following. See Pag. 160. Fig. 21.

Fortification and Gunnery are the two remaining tracts, which make up one Volume in this Translation,

tion, in which both those subjects are succinctly handled. In the one, the young Officer will see the maxims and methods of Military Architecture, with a proper explanation of Terms; and in the other, the Mathematical Principles applicable to Artillery, with just and necessary observations on the laws of motion with respect to Projectiles.

These being subjects much out of my own sphere, I therein consulted proper Judges: And must particularly own my obligations to Captain Richards of his Majesty's Train of Artillery, for his kind assistance with respect to the latter.

The two tracts on Navigation contained in the other Volume, are (especially the latter) uncommon Essays upon the subject: In the one is shewn the construction and use of Instruments, Charts, and Tables; with the application of Trigonometry to Sailing in the calculation of the Course, Distance, and Departure; also the necessary Addenda of finding the Prime, Epact, Golden Number, Moon's Age, and time of High Water in any Port, &c. In the other, which concerns the working of a Ship (a subject not treated on that I know of by any English Author) is considered and explained, from the establish'd laws of Motion, and the form and structure of a Vessel, the several motions a Ship is capable of receiving, and the proper methods of giving them; which necessarily led the Author into the consideration of the force and efficacy of the several Sails, according to their different situation; as also the power of the Rudder in changing the line of Direction. I have only to observe under this head, that as some of the practices he delivers differ from the methods used by

P R E F A C E

xi

the English, I have set you down the English manner of performance by way of Notes at the bottom of the page ; which likewise contain whatever other observations or remarks have occurred throughout the whole Performance.

In this part of the Work, I have been also careful to get proper assistance ; and am especially obliged to my good friend Captain Wilde, without whose Help it had been impossible for me to have understood the French terms, or to have express'd them with any tolerable propriety in English : But my friend being absent when the last hand was set to the Work, I must still bespeak the indulgence of the English Sailor, if he finds my expressions sometimes deviate from the true Sea dialect.

I have now done with our Author. As to the two Articles I have added, they are, First, An alphabetical explanation of such Sea terms and phrases as occur in the Work, which will not only be useful to such Gentlemen as are desirous to look into so curious a Theory, though not concern'd with the Sea ; but even necessary to those who undertake the study, with a view to their future practice. Secondly, An Appendix, containing the description and use of a compleat set of Pocket Instruments, wherein the little Apparatus, so necessary to every young Mathematician, is sufficiently explain'd ; and the several lines laid down upon the Plain Scale, Gunter Scale, and Sector, are considered, both with regard to their construction and application.

Having

Having thus laid before the Reader a summary account of the Work, I shall not farther take up his time with apologizing for the performance. I have done my best endeavour, both in the Translation and the additional Articles; I therefore submit the whole to the candid perusal of the Publick, well knowing that books of Science can only make their way by their Merit.



T H E

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THE CONTENTS.

TRACT the First,
The Elements of EUCLID.

BOOK	the first	Page 1
—	the second	19
—	the third	23
—	the fourth	30
—	the fifth	38
—	the sixth	45
—	the eleventh	56
—	the twelfth	63

TRACT the Second.

TRIGONOMETRY.

PART I.

The Construction of the Tables of Sines, Tangents,
and Secants 67

PART II.

Of Logarithms

75

PART

CONTENTS.

PART III.

The Resolution of right-lined Triangles Page 86

PART IV.

The Resolution of spherical Triangles 92

TRACT the Third, PRACTICAL GEOMETRY.

PART I.

Of Lines 121

PART II.

Of Surfaces 129

PART III.

Of Solids 136

PART IV.

Some Problems 139

TRACT the Fourth, MECHANICKS.

Definitions 143
Physical Suppositions 144
General Axiom 145
Of the Lever 147

CONTENTS.

Of inclined Planes	Page 148
Of Pullies	ibid
Of Capstons, Wheels, &c.	150
Of the Screw	151
Of Liquors	152
Of suspended Bodies	154
Of Asceleration	157

TRACT the Fifth,

FORTIFICATION.

Axioms	165
Definitions	166
Of laying down upon Paper the Plan of a regular Fortification	167
Of laying down Outworks upon Paper	171
Various Methods of Engineers	175
Of the Calculations of Fortifications	177
Of tracing out irregular Fortifications upon Paper	179
Of Marine Fortifications	184
Of tracing out Fortifications on the Ground	185
Of putting your Plan in Perspective orthographically	187

TRACT the Sixth,

GUNNERY.

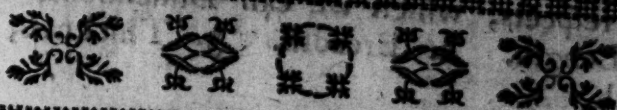
Definitions	189
Suppositions	191
Of the Proportions of the several Parts of a Cannon	192
Of the Cavity or Chase of a Cannon	195
Of the Outside of a Cannon	196
Of the Calibre, or Bore of a Cannon	198
Of the Bullet	200
Of	Of

CONTENTS.

<i>Of the Calibre of the Bullet</i>	200
<i>Of the Cause of the Motion of the Bullet in a Cannon</i>	202
<i>Of the Chamber of a Cannon</i>	205
<i>Of the Length of a Cannon</i>	ibid.
<i>Of the Charging a Cannon</i>	206
<i>Of a Cannon's Heating</i>	207
<i>Of the horizontal Carriage of a Cannon</i>	ibid.
<i>Of the Resistance of the Air</i>	210
<i>Of the Resistance of the Water, &c.</i>	214
<i>Of the instruments for finding the Elevation of a Cannon</i>	220
<i>Of the Force of the Ball against the Body it strikes</i>	222
<i>Of Mortars for Bombardments</i>	223



ELE.



ELEMENTS

OF

EUCCLID.

BOOK I.

DEFINITIONS.

1. **M**ATHEMATICKS are certain Sciences, by which we learn to compare things one with another, in regard to their extension, or to determine whether they are equal, or in what proportion one thing is greater, or less than another.

2. *Magnitude* therefore is that idea arising in our minds, from the comparison of one thing with another in regard to extension.

B

3. *Exten-*

3. *Extension* is to be considered in three different respects, which we call Dimensions, viz. 1. Length. 2. Breadth. 3. Thickness, or Depth.

4. A *Solid*, is a magnitude considered under three dimensions; as if the tonnage of a Ship is demanded, the Ship is considered as a *Solid*; and the question requires that we examine its length, its breadth, and its depth; for the longer, the broader, and the deeper it is, the greater will be its tonnage.

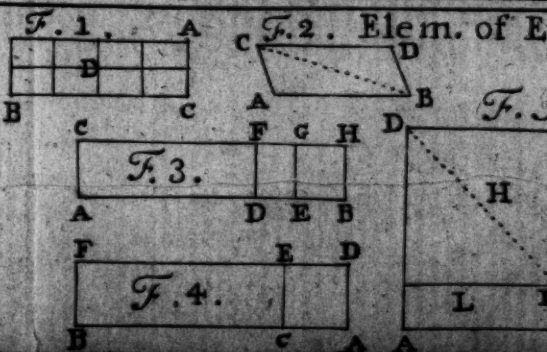
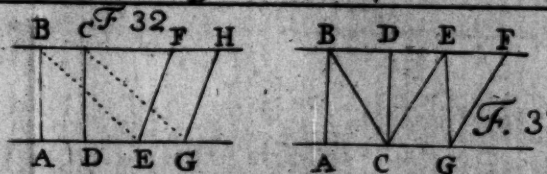
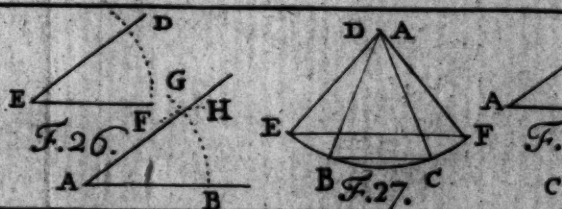
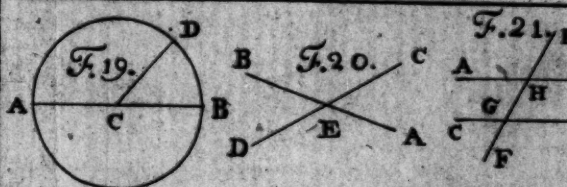
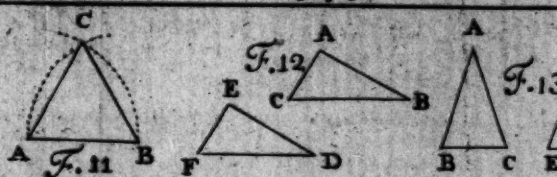
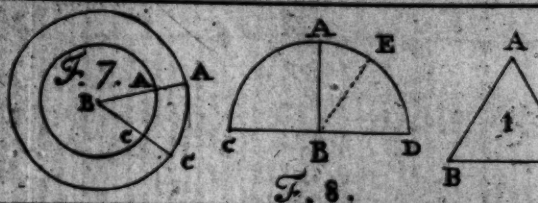
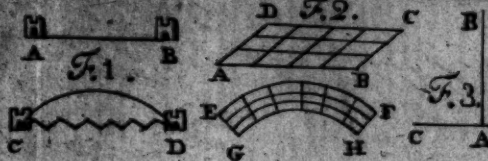
5. A *Superficies*, or *Surface*, is a magnitude comprehended under two dimensions, viz. length and breadth; as when we measure a field, we take it for a *Surface*, which the longer, and broader it is, the more acres it contains, without any regard to the depth, or thickness of the earth.

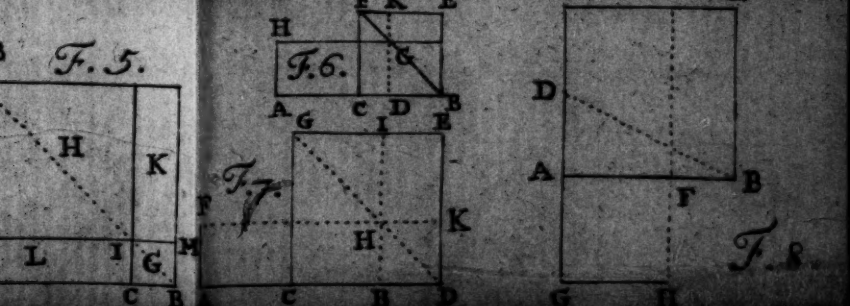
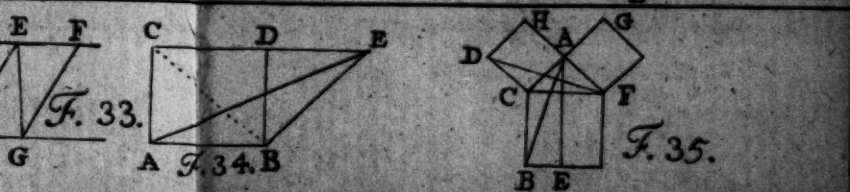
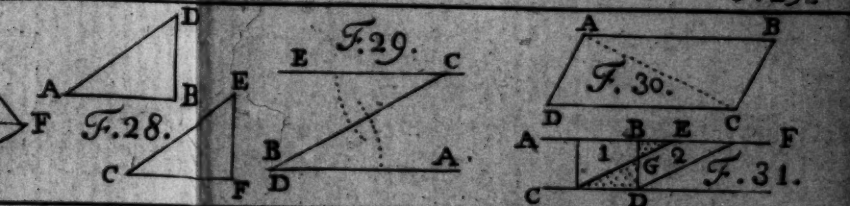
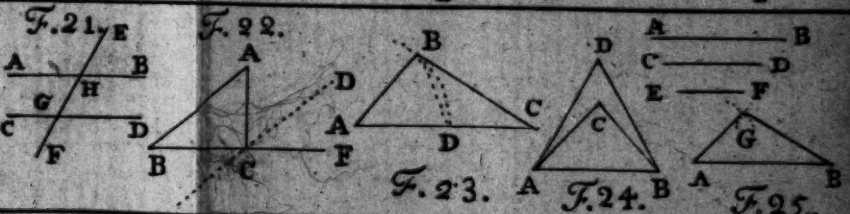
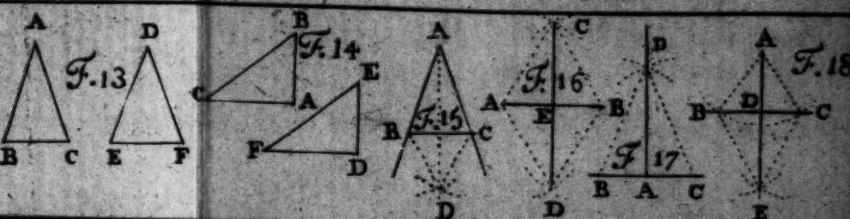
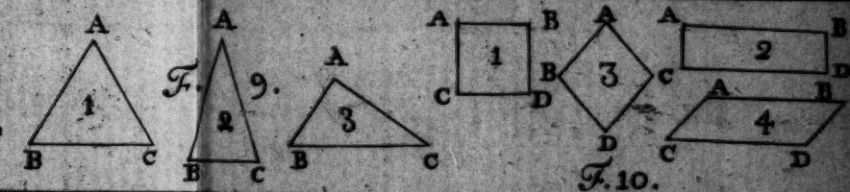
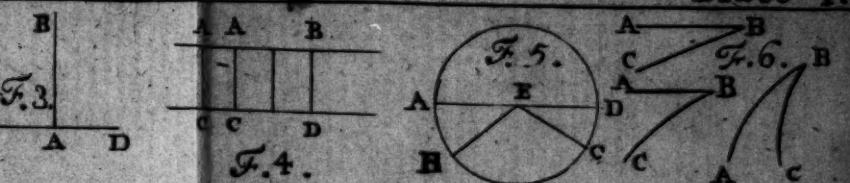
6. A *Line* is a magnitude considered only in length; thus when it is ask'd, how far one place is distant from another, we consider the space between the two places as a line, which, the longer it is, the farther distant are the places; but the breadth or thickness of the space, makes no difference in the distance of the places.

7. A *Point* is that in which we consider neither length, breadth, nor thickness, as in the foregoing instance, the two places are taken for two *Points*; for it is not at all necessary to know the length, breadth, or thickness of the two places to determine the distance between them.

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8. The extrems of a Line are Points: a Line being only a continuation of the motion of a Point. So Lines are the boundaries of a Superficies. In like manner also, Superficies are the boundaries of a solid body.

9. Two similar Surfaces are said to be equal, when the one being laid upon the other, perfectly covers it. In like manner, two similar Solids are accounted equal, when one being supposed to penetrate the other, is perfectly contain'd within the same limits.

10. A right Line, is the shortest distance between two Points: thus the Line AB is a right Line, as, on the contrary, the Line CD is curved, or crooked. *Fig. 1.*

11. A plane Surface is that upon which a right Line may be every way applied: Thus the Surface ABCD is plane, but the Surface EFGH curved, the hollow side of a curve Surface is called concave, the raised side convex. *Fig. 2.*

12. A Line is perpendicular to another, when it has no inclination on either side, as AB upon CD. *Fig. 3.*

13. The Lines AB and CD are parallel where all the perpendicular Lines between them are of equal length. *Fig. 4.*

B 2

14. A

14. A circle is a plane figure as ABC , whereof all the extremities are equally distant from

Fig. 5. one certain point. The circumference of a circle, is the curve line which encompasses it, as $A B C$. The center is the point E , in the middle. A right line drawn from the center to the circumference, is call'd a Radius, as BE ; and if, in the same manner, a right line be drawn quite cross the circle, passing thro' the center, it is called a Diameter, as AED : lastly, an arch is any part of the circumference, as BC .

15. A plane angle, is the distance or opening of two lines, which touch in one point,

Fig. 6. not making one line; thus the opening of the lines BA , BC , makes the angle ABC .

16. An angle is the greater, the wider the lines are apart: But as the length of the lines makes no alteration in their distance; so it makes no difference in the largeness, or smallness of the angle.

17. The measure of an angle, as ABC , is found by describing a circle round the

Fig. 7. point B , which circle being divided into 360 equal parts, called degrees, the number of such parts contained in the arch AC , is the quantity of the angle in degrees.

18. A right angle is that which is exactly measured by an arch of 90 degrees, or

Fig. 8. the fourth part of the circumference of a circle

a circle, as ABC . An *obtuse* angle is greater than a right angle, as ECB , and an *acute* angle less, as EBD .

19. A rectilinear angle is form'd by two right lines; a curvilinear by two curves; and a mix'd angle by one line right, *Fig. 6.* and one curve.

20. A Triangle is a plane Surface as ABC bounded by three lines. 1. If these lines are equal, the triangle is equilateral; *Fig. 9.* 2. If two of them are equal, it is an Isosceles triangle; 3. But if the three lines are all unequal, it is call'd a scalene triangle.

21. A Parallelogram is a plane bounded with four lines, whereof the opposite ones are parallel, as $ABCD$. There are *Fig. 10.* four sorts of Parallelograms: 1. The Square, which has all its sides and angles equal. 2. The long Square, which has its angles equal, but not its sides. 3. The Rhombus, which has its sides equal, but not its angles. 4. The Rhomboides, which has neither its sides, nor angles equal.

22. We call a Truth which we demonstrate, a *Proposition*. Of which there are several sorts: 1. *Theorems*, which are truths purely speculative. 2. *Problems*, wherein something is proposed to be done. 3. *Lemmas*, which are truths laid down only for the demonstration of others. 4. *Corollaries*, which are the consequences drawn from a proposition demonstrated.

23. *Axioms* are plain truths known to all the world; some of the principal of which are as follow :

1. The whole is greater than its part.
2. Two things are equal to each other, when each one is equal to the same third.
3. Two equal things equally augmented or diminished, will remain equal; but if they are augmented or diminished unequally, they will become unequal.

N. B. *The Propositions are numbered as they stand in Euclid.*

PROPOSITION I.

To make an equilateral triangle upon the line AB. From the point A with *Fig. 11.* the compasses open'd to the extent AB, describe the arch BC; and from the point B, with the same extent, describe the arch AC, which shall cut the arch BC, in the point C; then drawing the lines AC, and BC, each of them shall be equal to the line AB, (by *Def. 14.*) and to one another (by *Ax. 2.*) consequently the triangle ABC, will be equilateral, (according to *Def. 20.*)

PROPOSITION IV.

If the line AB of one triangle, be equal to the line ED of another triangle, and *Fig. 12.* also the line AC equal to the line EF. If, farther, the angle A of the

one, be equal to the angle E of the other, the two triangles are equal in every respect.

Demonstration. If the line ED, of the triangle EDF, be laid upon the line AB, of the triangle ABC, it will perfectly cover it, (by Def. 9.) and the line EF will fall upon the line AC; since then the angles A and E are equal, they will likewise exactly cover each other; so that the point F, will be found upon the point C, in the same manner as the point D, upon the point B; consequently the line DF, must also cover the line BC, and the whole triangle EDF, exactly cover the triangle ABC. Therefore it is equal in every respect, (by Def. 9.)

P R O P. V.

If the lines AB, AC, of the triangle ABC, are equal, the angles B and C shall be also equal. Make another triangle *Fig. 13.* DEF, exactly equal to the triangle ABC, so that the angle A, may be equal to the angle D, and the lines AB, and AC, equal to the lines DE, DF.

Demonstr. If the angle D, be so laid upon the angle A, that the line DE falls upon the line AB, the triangle DEF will perfectly cover the triangle ABC, and the angle E will be found equal to the angle B; but if the line DE be laid upon the line AC, then will the angle E be found equal to the angle C; the angles therefore B and C being each of them equal to the angle E, are consequently equal to one another, (by Ax. 2.)

P R O P. VI.

If the angles B, and C, of the triangle ABC are equal; the lines AB, AC are also equal. Suppose, as before, DEF, equal in every respect to ABC.

Demonstration. If the triangle DEF, be transferr'd upon the triangle ABC, by laying EF, upon BC, the one will perfectly cover the other; and if the point E, falls upon the point B, the line DE, will also be found equal to the line AB; but if the point E, be laid upon the point C, then will the line DE, be found equal to the line AC, and consequently the lines AB, AC, being each equal to the line DE, must also (by Ax. 2.) be equal one to the other.

P R O P. VIII.

If the lines AB, BC, of the triangle ABC, are equal to the lines DE, EF, of the triangle DEF, each to each, it is evident that the Base AC, cannot be equal to the Base DF, unless the angle B, be equal to the angle E: consequently (by Prop. 4.) the three lines of the triangle ABC, cannot be equal to the three lines of the triangle DEF, without the two triangles are equal in every respect.

P R O P. IX.

To divide an angle BAC into two equal parts:

Make the point A, a center, describe the arch BC, and thereon (by Prop. 1.) make an equilateral triangle BDC, so

shall

shall the line AD, drawn from the angular point
the opposite intersection, divide the angle
AC into two equal parts; that is to say, the
angle BAD, shall be equal to the angle CAD.

Demonstration. The triangle ABD, has all
sides equal to the sides of the triangle ACD;
consequently (by *Prop. 4.*) the angle BAD, must
be equal to the angle CAD.

PROP. X.

To divide the line AB, into two equal parts:
Take thereon (by *Prop. 1.*) the equi-
lateral triangles BDA, BCA, and *Fig. 16.*
Draw the line CD, which shall divide
the line AB in the point E, so that EB, and EA,
shall be equal.

Demonstration. The triangles ADE, BDE
have the side DE common, and the side BD is
equal to the side AD; (by *Def. 14.*) and farther,
the angle ADE, is equal to the angle BDE,
(by *Prop. 9.*) Therefore is the triangle ADE,
equal in every respect to the triangle BDE, (by
Prop. 4.) and the base AE, equal to the base
BE.

PROP. XI.

To raise from the Point A, a line perpendicu-
lar to the line BC: Make AB, AC
equal. From the points B and C, as *Fig. 17.*
centers, with the same opening of the
compasses describe two arches cutting each other
in D, draw the line DA, it shall be a perpendi-
cular; that is to say, the angles DAB, DAC,
shall

shall be equal, because (by *Prop. 8.*) all the sides of the triangle DAB, will be equal to all the sides of the triangle DAC.

P R O P. XII.

To let fall a perpendicular upon a line BC from a point given without it A. *Fig. 18.* A as a center describe an arch, which shall cut the line in the points B, and C, then making the equilateral triangle BEC, (*Prop. 1.*) the line AE shall be perpendicular to the line BC.

Demonstrat. Since the triangles ABD, and ACD, have the side AD common, and AB equal to AC, and the angle BAD being equal to the angle CAD, (by *Prop. 9.*) they are equal in every respect, (by *Prop. 4.*) and the angles D equal on each side; therefore is the line AE perpendicular to the line BC, (by *Def. 12.*)

P R O P. XIII.

The line DC falling upon the line AB, makes two angles viz. DCA, DCB, equal to two right ones: for if from the center C a circle be described, the semicircle ADB will exactly measure them, (by *Def. 18.*)

P R O P. XIV.

If the two angles (in the foregoing Figure) DCA, DCB, are equal to two right ones, two lines AC, BC, must consequently make one line, which is here the diameter of the circle ADB.

PROP. XV.

When two lines, as AB, CD , cut each other as in the point E , they form four angles, of which the opposite ones are equal, as BED, CEA , because either of them contains what is wanting in the angle BEC , to make two right ones, (by *Prop. 13.*) *Fig. 20.*

COROLLARY.

If the line EF , cuts the parallels AB, CD , the angle EHB , shall be equal to the angle CGF ; because the two parallels may be taken for one line, and then the two angles are opposite, as before, (by the preceding.) Farther, because the angle AHG is equal to the angle EHB , as the angle HGD to the angle CGF , being opposite; therefore the four angles EHB, AHG, HGD, CGF , shall all be equal, the angles CGF, EHB , are called outward alternates; the angles AHG, HGD inward alternates; the angles EHG, HGD alternately opposed. These three sorts of angles are then equal, when the two lines thus cross'd by a third are parallel: and when they are equal, the two lines thus cross'd by a third are parallels.

PROP. XVI. and XXXII.

If any side, as BC , of the triangle ABC , be produced, or drawn out, it will make the exterior angle ACF , equal to both the interior and opposite angles A and B . *Fig. 22.*

Demon-

Demonstration. Thro' the point C, let the line CD be drawn, parallel to the line AB; which will divide the outward angle into two parts of which the first, *viz.* ACD, will be equal to the angle A, and the second, *viz.* DCF, equal to the angle B, (by the preceding.)

COROLLARIES.

1. The three angles of a triangle are equal to two right angles: for the outward angle ACF with the adjoining angle ACB, are equal to two right ones (by *Prop.* 13.) therefore the angles A, B, with the same angle ACB, are equal to two right angles.

2. When therefore a triangle has one angle right, or obtuse, both the other angles must be acute.

3. If two angles of one triangle are equal to two angles of another triangle, their third angles are also equal.

PROP. XVIII.

If the side AB of the triangle ABC is shorter than the side AC, the angle C, opposite to the side AB, shall be less than the angle B, opposite to the side AC. Take AD, equal to the side AB, and draw BD.

Demonstration. The sides AB, AD, of the triangle ABD, being equal, the angles ABD, ADB, shall be equal, (by *Prop.* 5.) but the angle ADB, being an outward angle, is greater than the angle C, (by the preceding) therefore the angle ABD will be greater than the angle C, and consequently the angle ABC will be so much greater than the angle C.

PROP

PROP. XIX.

If the angle B , is greater than the angle C , the side AC , shall be greater than the side AB . *Fig. 23.*

• *Demonstrat.* If the side AC , was not greater than the side AB , either it would be equal, and the angle B would be equal to the angle C , (by *Prop. 5.*) or it would be less, and the angle C would be greater than the angle B , (by the preceding) both of which will be contrary to the supposition.

PROP. XX, XXI.

The two sides AC , CB , of a triangle, taken together, are longer than the third AB , which being a right line, is the shortest distance from the point A , to the point B . In the same manner the lines AD , DB , are longer than the lines AC , CB , because they are farther distant from the right line AB .

PROP. XXII.

To make a triangle of three given lines AB , CD , EF : First lay down a right line equal in extent to the line AB , then *Fig. 25.* open the compasses to the extent CD , and with one Foot in the point B , describe an arch; then taking the extent EF , between your compasses, with one foot in A , make another arch to cut the former in the point G ; so shall the triangle AGB , (by *Def. 14.*) be composed of the line AB , and the lines AG , BG , equal to the given lines AB , CD and EF .

PROP.

P R O P. XXIII.

To make on the point A, of the line AB, an angle equal to the angle DEF : From
Fig. 26. the point E, as a center, describe an arch DF, and from the point A, with the same extent of the compasses, describe another arch BG ; then opening the compasses, to the distance DF, from the point B, as a center, make an arch cutting the arch BG, in the point H, and the angle BAH, shall be equal to the angle DEF ; the two triangles being (by *Prop. 8.*) equal in every respect.

P R O P. XXIV.

If the sides DE, and DF, of one triangle, are of the same length with the sides CA
Fig. 27. and AB, of another : when those sides are farther distant than these, that is, if the angle D, is greater than the angle A, the base EF, will also be greater than the base BC, which is self-evident.

P R O P. XXV.

If the side AB, of the triangle ABD, is equal to the side CF, of the triangle CEF ;
Fig. 28. if also the angle A, be equal to the angle C, and the angle B to the angle F, the two triangles shall be equal in every respect.

Demonstration. If the triangle CEF, be laid upon the triangle ABD, in such manner, that the side CF falls exactly upon the side AB, then will

will the angle C agree with the angle A, and the angle F with the angle B; consequently the line CE, will fall upon the line AD, and the line FE, on the line BD, and the triangle CFE, perfectly cover the triangle ABD.

PROP. XXXI.

To draw thro' the point C, a line parallel to the line AB: Draw a line CD, cutting AB. On the point C, make the angle *Fig. 29.* DCE, equal to the angle CDA, (by *Prop. 23.*) and the line EC, shall be parallel to the line AB, (by *Cor. to Prop. 15.*)

PROP. XXXIII.

If the lines AB, DC, are equal, and parallel; the lines AD, BC, shall *Fig. 30.* be also equal, and parallel: Draw AC.

Demonstrat. The triangles ABC, ADC, are equal in every respect, (by *Prop. 4.*) because the side AC, is common to both, and the side AB, of the one, is supposed equal to the side CD of the other; and farther, the alternate inward angles BAC, DCA, are equal, (by *Cor. to Prop. 15.*) Therefore the base BC, must be equal to the base AD; and the angle ACB, equal to the angle CAD; and consequently BC, will be parallel to AD.

PROP. XXXIV.

If AB is parallel to DC, and AD parallel to BC, the opposite sides, and angles of the parallelogram ABCD, shall be equal. *Fig. 30.*
Demon-

Demonstration. The triangles ABC , ADC , having the base AC common, and the inward alternate angles BAC , DCA , CAD , ACB , equal, must be equal in every respect, (by *Prop.* 25.) therefore AB is equal to DC ; BC , equal to AD ; and the angle B , equal to the angle D .

P R O P. XXXV.

If the Parallelograms $ABCD$, $EFCD$, have the same base CD , and are between *Fig. 31.* the same parallels CD , AF , they shall themselves be equal.

Demonst. All the sides of the triangle ACE being equal to the sides of the triangle BDF , (by the preceding) the two triangles are equal (by *Prop.* 8.) Therefore if from each, the triangle BGE , which is common to both, be taken away, the space 1, which remains of the first, will be equal to the space 2, the remainder of the second; and consequently the space 1, with the triangle CDG , or the parallelogram $ABCD$, will be equal to the space 2, with the same triangle CDG , or to the parallelogram $EFCD$.

P R O P. XXXVI.

If the parallelograms $ABCD$, $EFGH$, which are between the same parallels AG , *Fig. 32.* BH , have their bases BC , EG , equal; they will each of them be equal to the parallelogram $BCGE$, (by the preceding) and consequently one to another,

P R O P.

P R O P. XXXVII.

If the triangles ABC , CEG , are between the same Parallels BF , AG ; and the base AC of the one, equal to the base CG *Fig. 33.* of the other, they are equal; because the one is the half of the parallelogram $ABDC$, and the other is the half of the parallelogram $CEFG$, (by *Prop. 34.*) which is equal to the parallelogram $ABDC$, (by the preceding.)

P R O P. XLI.

If the Triangle ABE , has the same Base AB as the parallelogram $ABDC$, and stands between the same parallels AB , CE , it shall be the half of the parallelogram $ABCD$; in the same manner as the triangle ABC , which (by the preceding,) is equal to it.

P R O P. XLVII.

If the angle A of the triangle AFC is right, the square of the side FC , which is opposite to it, shall be equal to the square of the side AC , and the square of the side AF . *Fig. 35.*

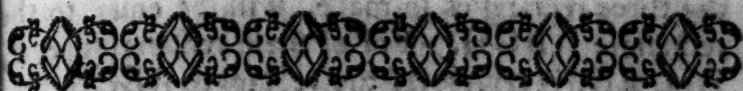
Suppose the three squares to be each made upon its proper side; the lines CA , GA , will then become one line, as will also the lines FA , HA , (by *Prop. 14.*) draw FD , and AB . I say, that the line AE , being drawn parallel to the line BC , shall divide the square FB , into two rectangular parallelograms; of which the one CE ,

CE, shall be equal to the square of the side AC; and the other FE, equal to the square of the side AF.

Demonst. The triangle FCD, having the same base CD, as the square DA, and being between the same parallels FA, CD, will be just equal to its half, (by *Prop.* 41.) in the same manner the triangle ABC, will be half the rectangle CE. But the triangle FCD, is equal to the triangle ABC, (by *Prop.* 4.) because the side CB is equal to the side CF, the side CA, to the side CD, and the angles ACB, FCD, are also equal, being each composed of one right, and the angle FCA. Therefore the half of the square AD, is equal to the half of the rectangle CE, and consequently the whole square AD, will be equal to the whole rectangle CE.

In the same manner it may be shewn, that the square FG, is equal to the rectangle FE.





BOOK II.

DEFINITIONS.

1. **A** *Rectangle*, is a Parallelogram, which has all its Angles right.

2. The area or Content of a rectangle, is the number of squares which it contains: Therefore to find the content, *Fig. 1.* we multiply the length by the breadth.

3. A rectangle is said to be contain'd under two lines, one of which expresses its length, the other its breadth. Thus *Fig. 1.* the rectangle D, is contain'd under the lines AC, BC.

4. The *Diameter* or *Diagonal* of a rectangle, is a line drawn from one angle to another which is opposite, as CB. *Fig. 2.*

PROP. I. and II.

If the side AB, of a rectangle ABCH, be divided into a certain number of parts, as suppose three, and upon each part be form'd a rectangle having the same *Fig. 3.*

same height AC , with the rectangle $ABCH$; the three rectangles so made, will be equal to the rectangle $ABCH$. For of these rectangles AF , DG , EH , is composed the rectangle $ABCH$.

P R O P. III.

If the line AB , be divided in the point C , and the perpendicular AD , be equal to one of those parts, as AC ; the rectangle $ABFD$, will be equal to the square of the part AC , to wit, $ADEC$; and the rectangle $CEFB$, which has the part AC for its breadth, and the other part CB for its length.

L E M M A.

If thro' the point I , of the diameter of a square DB , be drawn two lines parallel to the sides BA , DA ; there will be formed four rectangles, of which the two that are cross'd with the diagonal will be squares, to wit, G , and H .

Demonst. All the angles of the Parallelogram G , are right, (by 34. 1.) farther all its sides are equal; for in the right-angled, and isosceles triangle DBA , the equal angles B , and D , being (by 5. 1.) equal but to one right, are each of them half right angles. So in the right-angled triangle BIC , the angles I and B , being equal but to one right, the angle I , will be half a right angle, as the angle B ; and consequently the sides BC , CI will be equal (by 5. 1.) as also their opposites MI , MB ; and the rectangle G , will be a square.

P R O P.

PROP. IV.

If the line AB , be divided in the point C , the whole square of the line AB will be equal to the square of the part AC , and the square of the part BC , and two rectangles whose base shall be AC , and height BC ; that is to say, the squares G , and H , and the rectangles L , K , (by *Lemma* preceding.) *Fig. 5.*

PROP. V.

If the line AB , is divided equally in the point C , and unequally in the point D , the rectangle of the unequal parts AD , DB , with the square of the line CD , will be equal to the square of the part CB . *Fig. 6.*

Demonst. The line DG , or AH , is equal to the line DB , (by *Lemma* preceding) and the line BE , is equal to the line CA , by the supposition; therefore the rectangles DE , CH , are equal; and to each be added the rectangle DF , then will the whole square CE , be equal to the whole figure KDA , which is composed of the rectangle of the unequal parts DH , and of the square of the line CD , or FK .

PROP. VI.

If the line AB , be divided equally in the point C , and to it be annex'd the line BD ; the square of the line CD , will be equal to the rectangle KA , whose length is the whole line AD , and breadth the annex'd part BD , with the square HG , whose side is the line CB . *Fig. 7.*

Demonst.

Demonst. The line BD , or HK , is equal to the line FA , (by *Lemma* preceding) and the line AC , or CB , is equal to the line HI ; therefore the rectangle KI , is equal to the rectangle CF ; adding then to both, the figure $GCDKHI$, the rectangle KA , with the square GH , will be equal to the square CK .

PROP. XI.

If a square be form'd upon the line AB , and one of its sides, as AC , be equally divided in the point D : If, farther, the line DG , be drawn equal to the line BD ; the square $AGHF$, shall be equal to the rectangle FE .

Demonst. Since the line AC , is equally divided in the point D , and is lengthened by the line AG , the rectangle CH , with the square of the line AD , will be equal to the square of the line DG or DB , (by the preceding :) But the square AE , with the square of the line AD , is also equal to the square of the line DB , (by 47. 1.) Therefore the square AE , is equal to the rectangle CH ; taking then away from both the rectangle CF , the rectangle FE , will be equal to the square FG .





BOOK III.

DEFINITIONS.

1. **W**HEN one line meets another without cutting it, we say it touches, or is a *Tangent*: Thus the line AB touches, or is a *Tangent* to the circumference of the circle C . *Fig. 1.*

2. The *Segment* of a circle is a part cut off with a cord line: or it is a figure bounded by a right line, and an arch of circle; as ABC . *Fig. 2.*

3. A *Sector* is a part of a circle bounded by two radii, or semidiameters, and the included arch; as ABC . *Fig. 3.*

4. An angle is said to be in a segment, when the right line of the segment is taken for the base, and the angular point touches the arch: Thus the angle ABC , is said to be in the segment ABC . *Fig. 4.*

PROP.

P R O P. I.

If from the middle of the line BC ,
Fig. 5. be rais'd the perpendicular AF , it shall
 pass thro' the center of the circle FBC .

Demonst. If any Point E , not in the line AF ,
 be the center of the circle FBC , the triangles
 BEA , CEA , will be equal in every respect,
 (by 8. 1.) then would the line EA , be perpendicular
 to BC , and not FA , as was supposed.

P R O P. II.

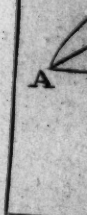
If from the center D of a circle,
Fig. 6. DB be drawn perpendicular to AC ,
 it shall divide the line AC into two
 equal parts BA , BC .

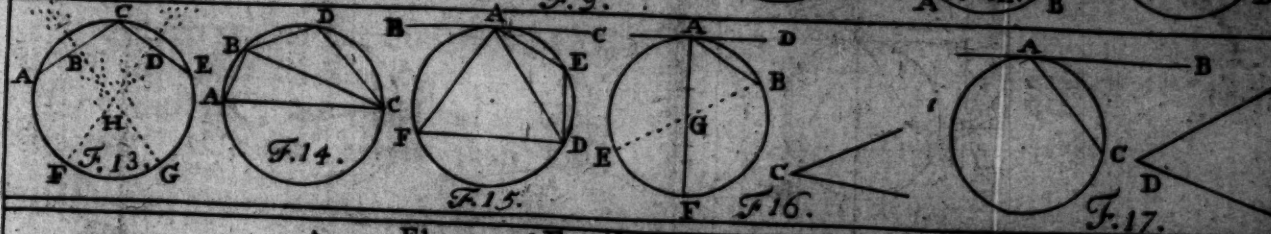
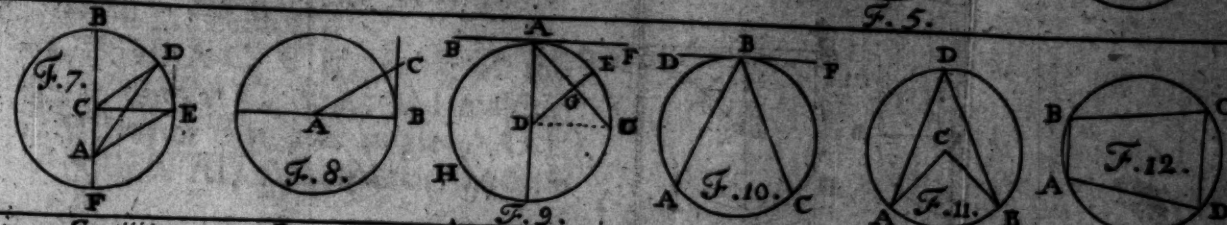
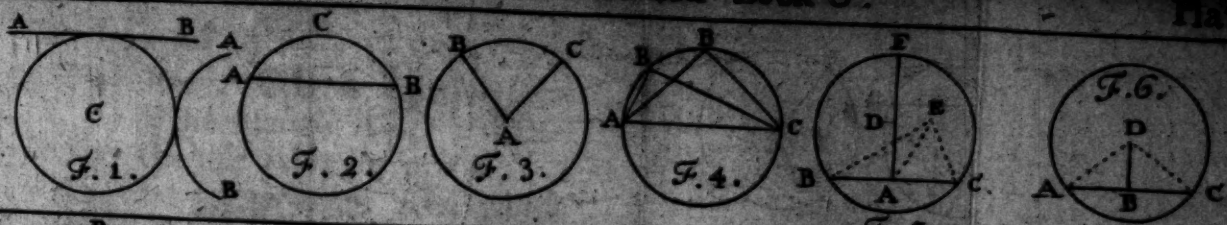
Demonst. In the triangles ABD , CBD , which
 are rectangled at the point B , the angles C , and A ,
 being equal, (by 5. 1.) their complements BDA ,
 BDC , will be also equal; and because BD is
 common to both triangles, and the side DA
 equal to the side DC . the two triangles shall be
 equal in every respect, (by 4. 1.) and the bases
 BA , BC equal.

P R O P. VII.

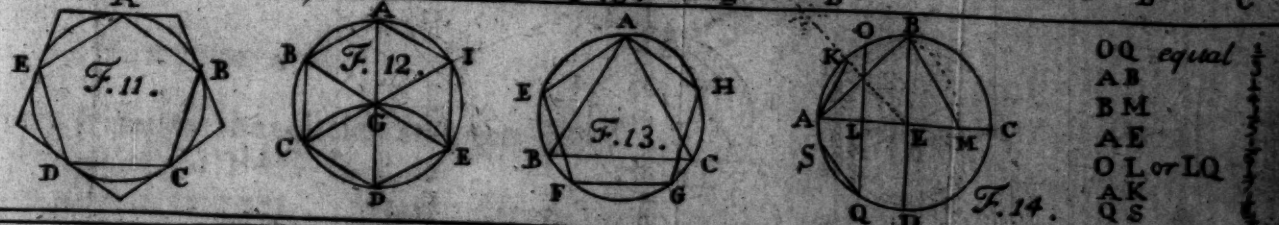
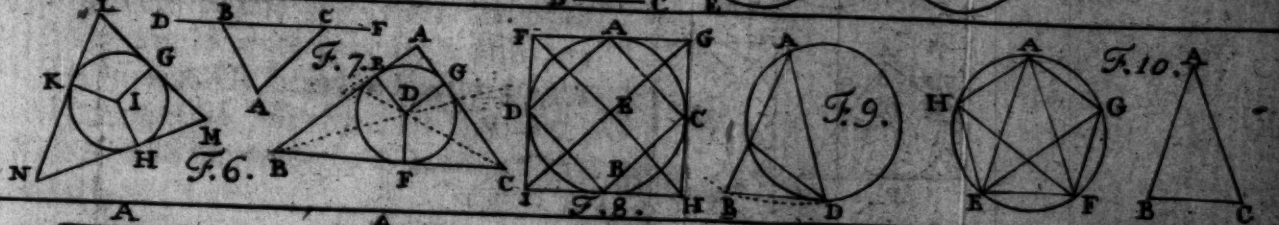
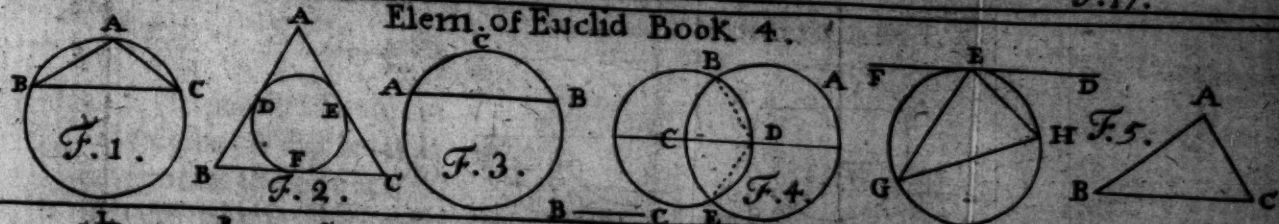
If from the point A , in a circle, of which it is
 not the center, several lines be drawn
Fig. 7. to the circumference; the line AB
 which passes thro' the center C , shall
 be longer than any other as AD , &c.

Demonst.



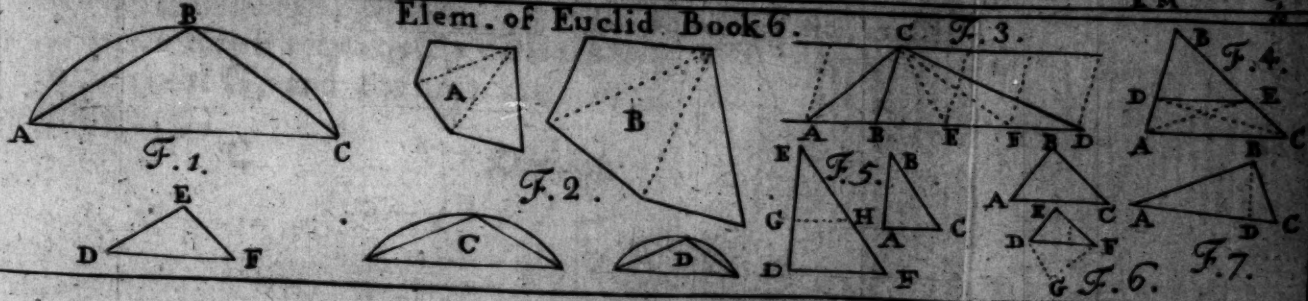


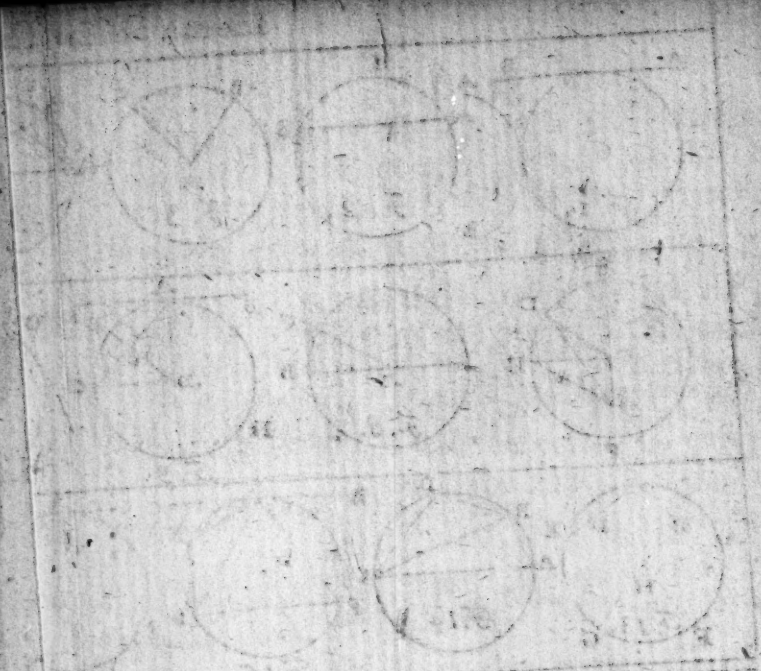
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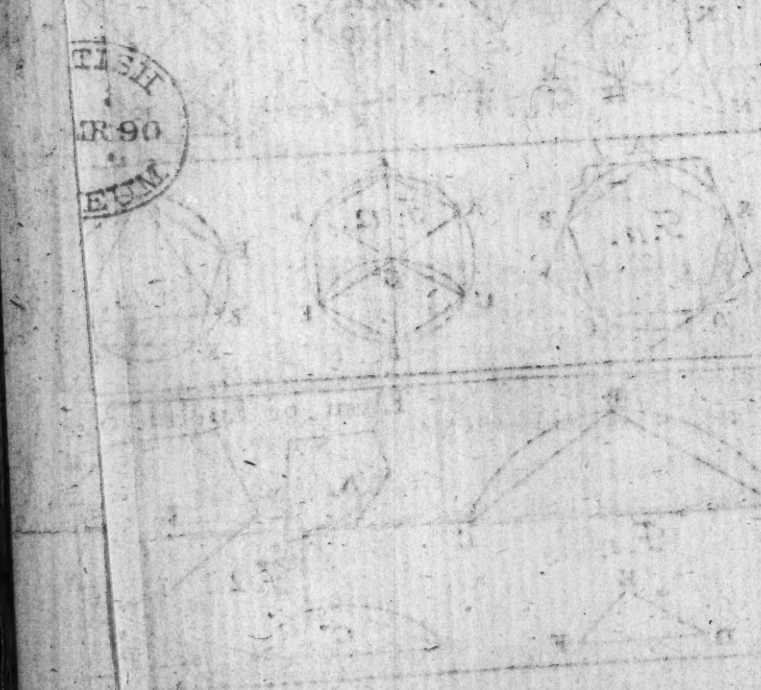
OQ equal
AB
BM
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OL or LQ
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EM

Elem. of Euclid Book 6.





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Demonstrat. The line ACB is equal to the two sides AC , CD , of the triangle ACD ; therefore it is longer than the side AD , (by 20. 1.)

Again, AD shall be longer than AE , which is farther distant from the center.

Demonst. The lines AC , CD , being of the same length as the lines AC , CE , and making the angle ACD greater than the angle ACE will also make the base AD greater than the base AE , (by 24. 1.)

Lastly, AF , which makes one line with ACB , shall be less than any other, as AE , &c.

Demonst. FA , with AC , are equal but to CE ; whereas AE , with AC , are longer than CE , (by 20. 1.)

PROP. XVI.

If the line BC is perpendicular upon the end of the diameter AB , it will touch, or be a tangent to the circle. *Fig. 8.*

Demonst. If it were to cut the circle, some of its points, as C , would enter, or be within the circle; and the line AC , being opposite to the right angle B , would be less than the line AB , which is opposite to an acute angle C ; which (by 8. 1.) cannot be.

LEMMA I.

If the line AB touch the circle, and the line AC cuts it; the angle BAC shall be measured by half the arch AC . Draw *Fig. 9.* from the center D , the line DE , perpendicular to CA ; so that it may equally divide the line AC , in the point G , and the arch AC , in the point E , (by 1. 3.)

C

Demonst.

Demonst. The angle ADG , with the angle DAG , are equal to one right, (by 16. 1.) and the angle BAC , with the same angle DAG , makes also one right, (by 16. 1.) therefore the angle BAC , is equal to the angle ADG , which is measured by EA , half the arch AC .

The angle CAF , shall be also measured by half the arch CHA ; because (by 16. 1.) the angles BAC , CAF , are together measured by half the whole circle.

LEMMA II.

The angle ABC , whose angular point touches the concave circumference of the circle, *Fig. 10.* is measured by half the arch AC , upon which it rests. Draw the tangent DBF .

Demonstrat. The three angles DBA , ABC , CBF , are measured by half the circle, (by 13. 1.) the angle DBA is measured by half the arch BA , and the angle CBF by half the arch CB , (by *Lemma* preceding) therefore the angle ABC shall be measured by half the remainder AC .

PROP. XX.

The angle ACB , at the center of a circle, is double the angle ADB , at the circumference; since (by *Lemma* preceding) the latter is measured by half, and the former by the whole arch AB .

PROP.

PROP. XXI.

Angles at the circumference insisting in the same arch are equal; each having for its measure (by Lemma preceding) half the arch upon which it rests.

PROP. XXII.

If the four angles of the quadrilateral figure ABCD, fall into the circumference of a circle, the opposite angles A, C, or *Fig. 12.* B, D, are equal to two right; because the angle A is measured by half the arch BCD, and the angle C, by half the arch BAD.

PROP. XXV.

To find the center of a circle whose circumference shall pass through three given points, A, C, E. Bisect the lines AC, *Fig. 13.* CE, in the points B, D, with the perpendiculars BG, DF, their point of intersection H, shall be the center of the circle; since (by 3.) both the lines BG, and DF, must pass through the center.

PROP. XXXI.

If an angle at the circumference ABC rests upon the semicircle AC, it is a right angle; but the angle BDC, which *Fig. 14.* rests upon the arch BAC, greater than a semicircle, is an obtuse angle; and the angle

C 2 CAB,

CAB, resting upon the arch BDC, less than a semicircle, is an acute angle; all which is evident from *Lemma 2.* preceding.

P R O P. XXXII.

If you draw the tangent BAC, and the line AD, to cut the circle in the point of
Fig. 15. contact, the angle CAD shall be equal to the angles of the alternate segment AFD, because (by *Lemma 2.*) both are measured by half the arch AD. In the same manner the angle BAD shall be equal to all the angles of the segment AED.

P R O P. XXXIII.

To describe upon the line AB an arch whose half shall be the measure of the angle
Fig. 16. C. Make the angle BAD equal to the angle C, (by 23. 1.) then having drawn AF, perpendicular to AD, make the angle ABE equal to the angle BAF, and the line BE shall cut the line AF, in the point G, which shall be the center of a circle, in which the equal lines GA, GB, shall be radii, (by 6. 1.) The line AD is a tangent (by 16. 3.) and half the arch AB the measure of the angle BAD, or the angle C, (by *Lemma 2.*)

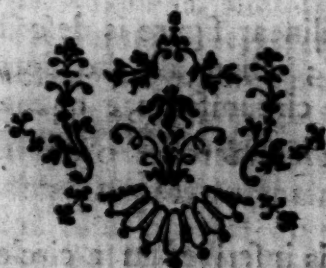
P R O P. XXXIV.

To cut off an arch AC, of a circle,
Fig. 17. such, that its half shall be the just measure of a given angle D; or having a
 circle

circle given, to cut from it a segment capable to contain a certain angle. Draw the tangent A B, make the angle BAC equal to the angle D, and half the arch AC shall be the measure thereof.

P R O P. XXXV, XXXVI.

Shall serve as *Corollaries* to the 16th of the 6th Book.



BOOK IV.

DEFINITIONS.

1. **A** Rectilinear figure is said to be inscribed in a circle ; when all the angular points of the former fall into the circumference of the latter. Thus the triangle ABC is inscribed in the circle ABCD.

2. A rectilinear figure is described about a circle ; when all the sides thereof touch the circumference of the circle. Thus the triangle ABC is described about the circle DEF.

3. A line is inscribed in a circle, when its extremes touch the circumference. Thus the line AB is inscribed in the circle ABC.

PROP. I.

To inscribe in a circle AEBD a line BC, not exceeding its diameter ; take between the feet of your compasses the length of the line BC, and setting one foot

at the extremity of the diameter, describe a circle cutting the circle $AEBD$ in D , and E ; then draw the line BD , or BE , which will be equal to BC , (by *Def.* 14. 1.)

PROP. II.

To inscribe in a circle a triangle EGH , equiangular to another triangle ABC . Draw the tangent FD , and at the point of contact E make the angle DEH equal to the angle B , and the angle $FE G$ equal to the angle C , (by 23. 1.) then draw the line GH , and the triangle EGH will be equiangular to the triangle ABC .

Demonst. The angles DEH is equal to the angle EGH of the alternate segment, (by 32. 3.) but the angle DEH is equal to the angle B , and consequently the angles B and G are equal, (by *Ax.* 2.) by the same reason the angles C and H are also equal; then (by *Cor.* 2. of 16. or 32. of 1.) the angles A and GEH will be equal: therefore the triangles EGH and ABC are equiangular.

PROP. III.

To describe a triangle LMN , about a circle GHK , equiangular to another triangle ABC . Produce one side of the given triangle, as BC , towards D , and F ; then make the angle $G I H$ equal to the angle ABD and the angle $H I K$ equal to the angle ACF , draw the tangents $LG M$, LKN , and NHM , through the points G , K , H , and they shall concur in angles equal to the angles given.

Demonstrat. All the angles of the quadrilateral $G I H M$ are equal to four right angles, because it may be divided into two triangles. The angles IGM and IHM , which are made by the tangents, are right angles; therefore the angles M and I are equal to two right angles, as are also the angles $A B C$, and $A B D$: but the angle GIH , is equal to the angle ABD : therefore the angle M , will be equal to the angle ABC . By the same reason the angles N and $A C B$ are equal; and therefore the triangles $L M N$ and $A B C$ are equiangular (by *Corol.* 2. of 16. or 32. 1.)

P R O P. IV.

To inscribe a circle EFG , in a triangle ABC

Bisect the angles B , and C , (by the 9. 1.)

Fig. 7. draw the lines BD , and CD , which will concur in the point D ; from this point draw the perpendiculars DE , DF , and DG , which will be equal, (by *Def.* 14. 1.); therefore a circle described from the center D , with the distance DE , will pass through F and G .

Demonstrat. The triangles DEB and DFB have the angles E and F equal, being both right angles; the angles DBE and DBF are also equal, and the side DB common: therefore the triangles will be equal in all respects, (by 25. 1.) and the sides DE and DF equal. In the same manner may the sides DF and DG be proved equal. A circle may then be drawn, which shall pass through the points E, F, G ; and because the angles E, F , and G , are right angles, the sides AB, AC , and BC , will touch the circle, which will consequently be inscribed in the triangle.

P R O P.

PROP. V.

To describe a circle about a triangle. See *Prop. 25. 3.*

PROP. VI. VII.

To inscribe a square in a circle ACBD, or to describe a square about the said circle.

Having drawn the two diameters AB *Fig. 8.* and CD, cutting each other perpendicularly at the center E; for the inscribed square, draw the lines AC, CB, BD, DA, and it is done: and for the circumscribed square, it is made by drawing the tangents FG, GH, HI, and IF, all which is evident.

PROP. VIII.

To inscribe a circle in a square. Bisect the several sides, and the point E, *v. Fig. 8.* where the lines concur, will be the center.

PROP. IX.

To describe a circle about a square. Draw the diagonals AB and CD, and *v. Fig. 8.* the point E, where they cut each other, will be the center.

P R O P. X.

To describe an isosceles triangle ABD , having the base angles B and D , each double to the angle A . Divide the line AB , (by 11. 2.) so that the square of AC may be equal to the rectangle under AB and BC ; and from the center A , at the distance AB , describe the arch BD , in which draw BD equal to AC ; then drawing the line DC , describe a circle about the triangle ACD , (by Prop. 5.)

Demonstration. Since the square of AC , and BD , is equal to the rectangle of AB and BC , and the line BD touches the circle ACD , at the point D ; therefore the angle BDC will be equal to the angle A , comprehended in the alternate segment CAD , (by 32. 3.) Now the angle BCD , being an external angle in respect of the triangle ACD , is equal to the angles A , and CDA ; therefore the angle BCD is equal to the angle BDA . Farther, the angle ADB is equal to the angle ABD , (by 5. 1.) therefore DCB and DBC are equal, and (by 6. 1.) the sides BD and DC will be equal; and since BD is equal to AC , the sides AC and CD will be equal, (by Ax. 2.) and so likewise the angles A and CDA ; therefore the angle ADB , is double the angle A .

P R O P. XI.

To inscribe a regular Pentagon in a circle. Describe (by the preceding) an isosceles triangle ABC , having each angle

B, and C at the base double the angle A. Then (by *Prop. 2.*) inscribe within the circle the triangle DEF, equiangular to ABC; next divide the angles DEF, and DFE, each into two equal parts and draw the lines EG, and FH; lastly, join the lines DH, DG, GF, EH, and you will have described a regular Pentagon, that is to say, a Pentagon having equal sides, and equal angles.

Demonst. The angles DEG, GEF, DFH, and HFE, being the halves of the angles DEF, and DFE, each of which is double to the angle EDF, are equal to the angle EDF; and consequently the five arches upon which they rest, are equal, (by 21. 3.) secondly, the angles DGF, GFE, &c. having each three of those equal arches for its base, will be also equal, (by Ax. 3.) therefore the sides and angles of the pentagon DHEFG, are equal.

P R O P. XII.

To describe a pentagon about a circle. Inscribe a regular pentagon ABCDE, in the circle, (by the preceding) and drawing *Fig. 11.* tangents thro' the points A, B, C, D, E, (by the 16. 3.) you will have described a regular pentagon about the circle.

P R O P. XIII. XIV.

To inscribe a circle in any regular Polygon; or to describe a circle about a regular Polygon. Bisect any two sides or two angles not opposite, and where the lines concur shall be the center.

P R O P.

PROP. XV.

To inscribe a regular Hexagon in the circle
 $ABCDEF$; draw the diameter
Fig. 12. AD , then with the compasses open'd
 to the radius of the circle, set one foot
 in the point D , and describe the arch CE , then
 draw the diameters EGB , and CGF ; and the
 lines AB , AF , FE , &c. shall form the Hexagon
 required.

Demonstration. 'Tis evident, that the triangles
 CDG , and DGE , are equilateral; therefore
 the angles CGD , DGE , and those oppos'd to
 them at the top BGA , and AGF , are each of
 them the third part of two right angles; that is
 to say, contain 60 degrees each. Now all the
 angles that can be made about the same point, are
 equal but to four right angles, or 360 degrees;
 therefore taking away four times 60, that is, 240,
 from 360, there will remain 120 for BGC , and
 FGE , which therefore each contain 60 degrees;
 consequently all the angles at the center being
 equal, all the arches and all the sides will be
 equal; and every angle, as A , B , C , &c. will be
 compounded of two angles of 60 degrees each,
 that is 120 degrees, and therefore will be equal.

COROLLARY.

The side of a hexagon is equal to the semi-
 diameter.

PROP.

PROP. XVI.

To inscribe a regular Pentedecagon in a circle.
 Inscribe in the circle an equilateral triangle, (by *Prop. 2.*) and a regular pentagon, (by *Prop. 11.*) so that the angles may meet at the point A, the line BF, will be the side of a Pentedecagon. *Fig. 13.*

Demonst. Since the line AB is the side of an equilateral triangle, the arch AEB will be the third part of the whole circle, that is, five fifteenths. But the arch AE, equal EF, being the fifth part, will contain three fifteenths, consequently the arch BF, being one third of EF, will be one fifteenth.

As a Supplement to this Fourth Book of Euclid, may be added this practical method, to divide a circle, as ABCD, into any number of equal parts, not exceeding ten.

Describe a circle, whose center let be E, and cross it with two diameters AC, and BD, at right angles to each other; *Fig. 14.* and set off from the point A, AO, and AQ, equal to BE the radius, and join OQ, so shall OQ be the chord of the third part of the circle; then join AB, which line AB will be the chord of the fourth part; upon L, with the distance LB, describe the arch BM, and join BM, which line shall be the fifth part; AE, or BE the radius, is the sixth part; and LO, or LQ, the seventh part; bisect the angle AEB, and KA will be the eighth part; divide the arch QAO, into three parts at S, and join SQ, which will be the ninth part; lastly, EM will be the chord of the tenth part.

BOOK

BOOK V.

DEFINITIONS.

1. **A** Part is a lesser quantity compared with a greater; thus a line of two feet is a part of one of five.

2. A Quantity which is exactly contain'd a certain number of times in another is call'd an aliquot part: thus a line of two feet is an aliquot part of one of four feet; but is term'd an aliquot part of one of five feet. An aliquot part takes its particular name from the number of times it contain'd in the quantity of which it is a part: when it is contain'd twice, we call it a half; thrice, a third; if four times, a fourth, &c.

3. The Ratio of one magnitude to another, the relation that the one has to the other, with regard to the number of times it contains any of its aliquot parts: thus the Ratio of a line of two feet with one of six, is the relation or respect it has to a line of six feet, in that it once contains its third part, twice its sixth, and four times its twelfth, &c.

4. The

4. There are two sorts of Ratios : 1. When one magnitude exactly contains a certain number of times an aliquot part of another, as a line of two feet contains the third part of a line of six feet exactly once, in such case the ratio is call'd rational. 2. When one magnitude does not exactly contain any number of times an aliquot part of another ; as it happens with numbers or quantities which are incommensurable. For example, the diagonal of a square is something less than three times the half of its side, and something more than five times the fourth part, &c. without being the exact measure of any part : in such case the ratio is irrational.

5. In every Ratio there are two terms, the first of which is call'd the Antecedent, the other, the Consequent : Thus in the Ratio of the magnitude A, to the magnitude B, A is the Antecedent, and B the Consequent ; on the contrary, if the Ratio is express'd, as B to A, then is B the Antecedent, and A the Consequent.

6. If the antecedent exactly contains its consequent a certain number of times, the Ratio is call'd multiple, and is said to be double, triple, or quadruple, &c. according as the antecedent contains its consequent twice, thrice, or four times, &c. but if, on the contrary, the antecedent is exactly contained a certain number of times in its consequent, the Ratio is call'd sub-multiple, and is express'd according to the number of times, by the words sub-double, sub-triple, or sub-quadruple, &c.

7. We

4. The

7. We express the value of a rational Ratio by the figure shewing the number of times the antecedent contains its consequent, or one of its aliquot parts; and these figures we call Exponents: thus 2 is the Exponent of double Ratio, and $\frac{1}{2}$ is the Exponent of sub-double Ratio. In the same manner $\frac{3}{5}$ is the Exponent of a Ratio, wherein the antecedent contains three times the fifth part of its consequent.

8. Two Ratios, whether rational or irrational, are said to be equal, when the antecedent of the one contains any aliquot part of its consequent for many times (whether exactly or not) as the antecedent of the other contains the same part of its consequent: Thus not only two Ratios double or sub-double, triple, or sub-triple, are equal, and also all rational Ratios which have the same Exponent; but even the irrational Ratios A and B shall be equal, if the antecedent of the Ratio A contains the third part of its consequent something less than six times, and its sixth part something more than eleven times, &c. and the antecedent of the Ratio B, in the same manner contains something less than six times the third part of its consequent, and something more than eleven times its sixth part; and that the same may be said of all the other parts both of the one, and the other consequent.

9. Proportion is an equality of Ratios: Thus we say, there is a proportion between the magnitudes or quantities $\left\{ \begin{matrix} 4 & 2 & 12 & 6 \\ A, & B, & C, & D \end{matrix} \right\}$ if the Ratio

the quantity 4 (A) to the quantity 2 (B) is equal to the Ratio of the quantity 12 (C) to the quantity 6 (D) which is usually express'd in the following manner, $A : B :: C : D$, that is, as A is to B, so is C to D. Proportion requires at least three Quantities ; for there cannot be two Ratios without at least three Quantities.

PROP. VII.

If the quantities A and B are equal, they will have an equal ratio to any third quantity, as C ; for the quantity A will contain every part of C, as many times as the quantity B will contain the same. In like manner the quantity C will have the same ratio to both the quantities A and B, because it will contain any part of the one as many times as it will contain the same part of the other.

PROP. VIII.

If A is greater than B, it will have a greater ratio to C, because it will oftener contain any of its parts. On the contrary, C will have a less ratio to A than to B, because the parts of A being greater than those of B, they will be fewer times contain'd in C.

PROP. IX.

If the two quantities A and B have the same ratio to a third as C ; or if C has the same ratio to A as to B ; the quantities A and B are equal.

PROP.

PROP. X.

If A has a greater ratio to C than B; or if C has a less ratio to A than to B; A must be greater than B.

These two Propositions are demonstrated as the foregoing.

PROP. XI.

It is evident, that two ratios are equal, if each is equal to a third.

PROP. XII.

If $A : B :: C : D$, then by conjunction $A + C : B + D :: A : B$.

Demon. If A contains twice the third part of B: C will contain twice the third part of D, and consequently A and C will contain twice the third part of B and D.

PROP. XIII.

If $A : B :: C : D$, and A has a greater ratio to B, than E to F, it is evident, that C will have a greater ratio to D, than E to F.

PROP. XV.

It is also evident, that A is to B, as the half of A to the half of B, the fourth of A to the fourth of B, and so to the other aliquot parts of A, and of B.

PROP. XVI.

If $A : B :: C : D$, then by exchange, $A : C :: B : D$, supposing these four quantities of the same kind, that is all lines, or all superficies; or all solids.

Demonst. If A is $\frac{2}{3}$ of B , then will C also be $\frac{2}{3}$ of D ; seeing therefore $\frac{2}{3}$ of B , are to $\frac{2}{3}$ of D , as A to D , (by the preceding) it may be concluded that $A : C :: B : D$.

PROP. XVII.

If $A : B :: C : D$, then by division A less $B : C$ less $D :: D : D$.

Demonst. If A contains $\frac{5}{3}$ of B , C will also contain $\frac{5}{3}$ of D ; then as A less B , is equal to $\frac{2}{3}$ of B ; so C less D , is equal to $\frac{2}{3}$ of D ; but if A is less than B , A less B will be a negative quantity, and consequently D must also be greater than C .

PROP. XVIII.

If $A : B :: C : D$, then by composition A more $B :: C$ more $D :: D$.

Demonst. If A contains $\frac{5}{3}$ of B , C will contain $\frac{5}{3}$ of D ; then as A more B , will contain $\frac{8}{3}$ of B ; C more D , will contain $\frac{8}{3}$ of D .

PROP.

PROP. XXII.

If $A : B :: C : D$, and $B : E :: D : F$, then equality of ratio, $A : E :: C : F$.

Demonst. If A is the third part of B , C will be the third part of D ; and if B is the fourth part of E , D will be the fourth part of F ; therefore as A will be the third part of $\frac{1}{4}$ of E , so C will be the $\frac{1}{3}$ of $\frac{1}{4}$ of F .

PROP. XXIII.

If $A : B :: E : F$, and $B : D :: C : E$, then interrupted ratio, $A : D :: C : F$.

Demonst. If A is $\frac{1}{3}$ of B , E will be $\frac{1}{3}$ of F ; and if B is $\frac{1}{4}$ of D , C will be $\frac{1}{4}$ of E ; then A will be $\frac{1}{3}$ of $\frac{1}{4}$, or $\frac{1}{12}$ of D ; so C will be $\frac{1}{4}$ of $\frac{1}{12}$ of F .

PROP. the Last.

If $A : B :: C : D$, then by inverted ratio, $A :: D : C$.

Demonstrat. If A is $\frac{1}{4}$ of B , C will be $\frac{1}{4}$ of D ; therefore as B will be four times A , so D will be four times C .

N. B. The preceding demonstrations are universal, tho' they seem appropriated to particular cases; and may in the same manner be applied to all cases, both for commensurable and incommensurable quantities; for what is here said of thirds and fourth parts, may be as well observed of any aliquot parts.

B O O K VI.

D E F I N I T I O N S.

A Rectilinear figure is said to be inscribed in a curvilinear, when all the angles of the former fall exactly into the boundaries of the latter; a rectilinear figure circumscribes a curvilinear, when all the lines of the first, if it be a surface, or its planes, if a solid, touch those of the last.

2. If all the angles of the triangle $A B C$ are equal to the angles of the triangle $D E F$, that is, if the angle A be equal to the angle D ; the angle B to E ; and the angle C to F ; the two triangles are call'd similar. As other figures, they are said to be similar, when the triangles of which the one is composed, are similar to the triangles which make up the other. Thus the rectilinear figure A is similar to the rectilinear figure B , if the three angles which compose the figure A , be similar to the three triangles of the figure B : circular segments as $C D$ are similar when they contain equal angles C and D .

3. Homo-

3. Homologous sides, are those which in similar triangles are opposite to equal angles.

Fig. 1. Thus the angle A being equal to the angle D, the side C D will be homologous to the side E F.

4. A ratio is said to be compounded of two others, when its Index or Exponent is the product of those two exponents multiplied together. Thus a sextuple ratio is compounded of double and triple ratio; for 2, the exponent of double ratio, multiplied by 3, the exponent of triple ratio, produces 6, the exponent of sextuple ratio.

5. If the exponent of any ratio be multiplied into itself, it will produce a ratio double or duplicate to the first. Thus quadruplicate ratio is the double of double ratio; because multiplying 2, the exponent of double ratio, by itself, it produces 4, the exponent of quadruple ratio. So also the ratio whose exponent is $\frac{1}{4}$, is double or duplicate to the ratio whose exponent is $\frac{1}{2}$.

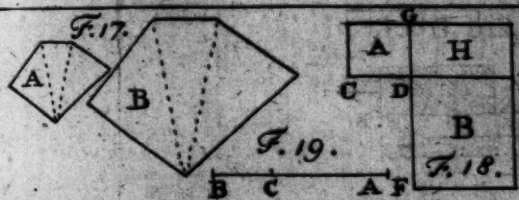
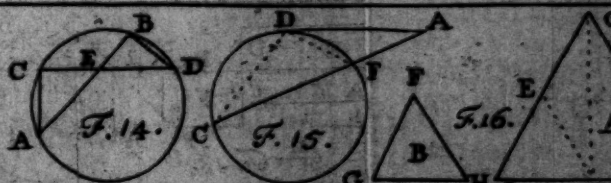
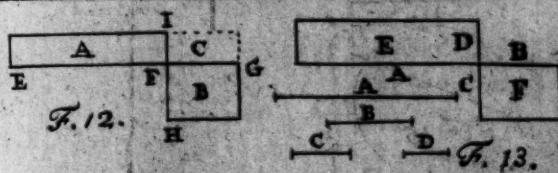
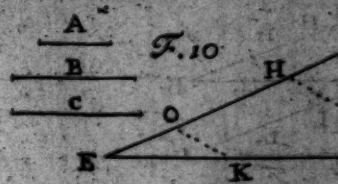
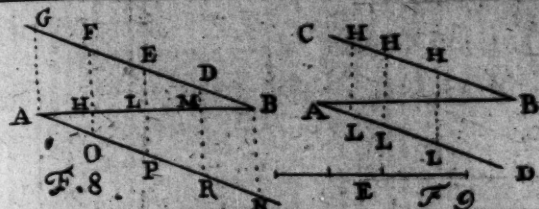
6. If the exponent of any ratio be multiplied by the exponent of its double or duplicate ratio, the product will be its triple or triplicate ratio. Thus the ratio, whose exponent is 8, will be triple to the ratio whose exponent is 2. In like manner the ratio whose exponent is $\frac{1}{27}$, will be triple to the ratio whose exponent is $\frac{1}{3}$.

L E M M A.

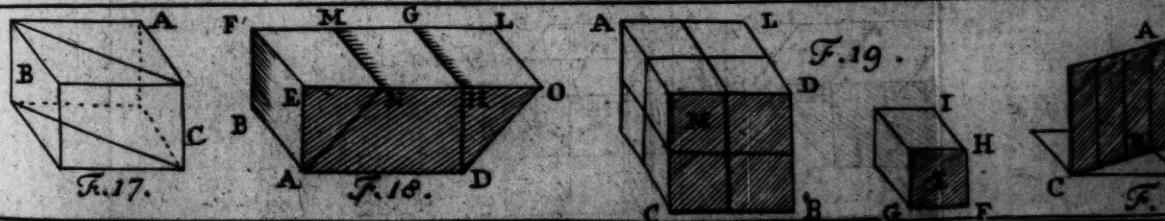
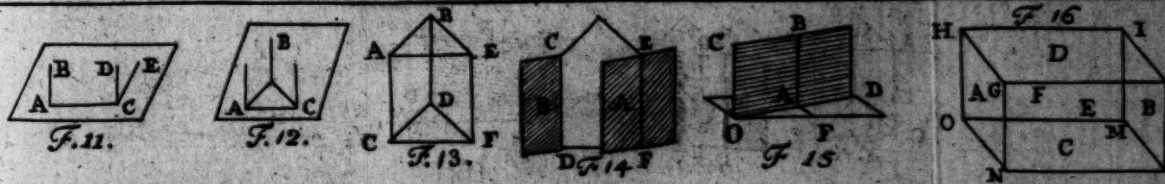
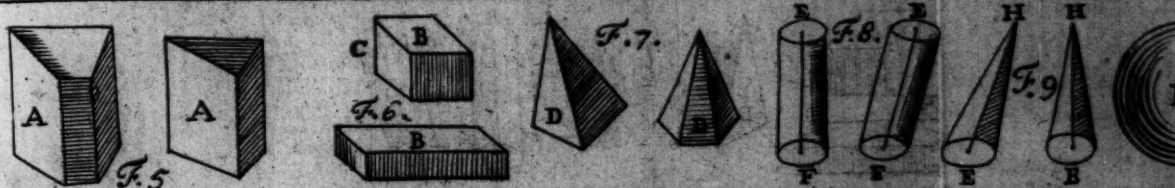
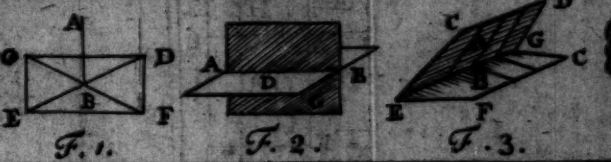
In a succession of quantities, as A, B, C, the ratio of the first to the last will be composed of the intermediate ratios; that is, the ratio of A to C will



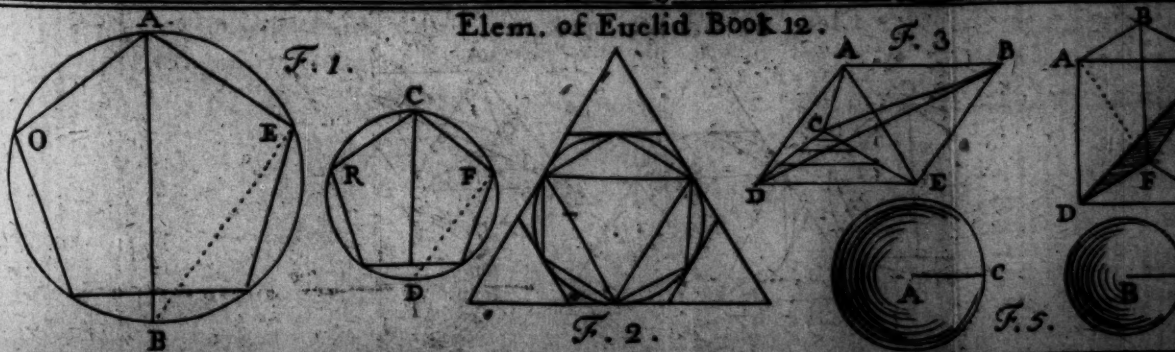
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Elem. of Euclid Book 12.





be compounded of the ratio of A to B, and B to C.

Demonst. If A is the third part of B, and B fourth part of C; A will be the $\frac{1}{3}$ of $\frac{1}{4}$, that is of C.

C O R O L L A R Y.

from whence it follows, that if several magnitudes as A, B, C, D, E, have a common ratio; it is to say, if $A : B :: B : C$, and $B : C :: C : D$, &c. the ratio of A to C, will be double ratio of A to B; and the ratio of A to D will triple the ratio of A to B; because the ratio of C to D is compounded of the ratios of A to B, and of B to C; and the ratio of A to D is compounded of the ratios of A to C, and of C to D, or of A to B.

P R O P. I.

If the triangles A B C, B C D, are of the same height, that is to say, stand between the same parallels, the triangle A B C *Fig. 3.* will be to the triangle B C D, as the base A B to the base B D.

Demonst. Suppose the base A B, to be but a third part of the base B D, then drawing the lines C E, C F, to the points E, F, they shall divide the line B D into three equal parts, and it will appear, that the triangle A B C will be equal to the triangle B C E, (by 37, 1.) which is but a third part of the triangle B C D.

In the same manner, it is demonstrable, that if the base A B were but the fourth part of the base B D, the triangle A B C would be but a fourth part of the triangle B C D.

part of the triangle BCD, &c. The same proof extends likewise to Parallelograms.

PROP. II.

If the line DE be drawn parallel to the base AC, of the triangle ABC, it will cut the sides AB, BC, proportionally, that is to say, the part AD will be to the part BD, as the part CE to the part EB. Draw AE and DC.

Demonstration. The triangles DEA, DEC, having the same base DE, and being between the same parallels CE, AC, are equal, (by 27.) therefore the triangle ADE will be to the triangle DEB, as the triangle DEC to the triangle DEB, (by 7. 5.) but the triangle ADE is to the triangle DEB, as AD to DB, (by 1. 6.) as is likewise the triangle CDE to the triangle DEB, as CE to EB; therefore $AD : DB :: CE : EB$.

PROP. IV.

If the triangle ABC is similar to the triangle DEF, the side DE will be to the side AB, which is homologous to it, as the side FE to its homologous side CB.

Fig. 5. Upon ED take the line EG, equal to AB; upon EF take EH, equal to BC; and draw GH.

Demonst. The triangle EGH, will be equal in every respect to the triangle ABC, (by 4. 1.) and the angle G will be equal to the angle A, to the angle D; therefore (by Cor. to 15. 1.) the line GH will be parallel to the line DF, and

consequently (by the preceding) $DG : GE :: FH : HE$; then by composition, (by 18. 5.) $DE : GE$, or $AB :: FE : HE$, or BC .

PROP. IV.

If the sides of the triangle ABC are proportional to the sides of the triangle DEF ; that is to say, if as $AB : DE :: BC : EF$ and $AC : DF$, the two triangles are equiangular. Upon DF describe the triangle DGF , so that the angle D may be equal to the angle A , and the angle F to the angle C . *Fig. 6.*

Demonst. Since the triangle DFG is similar to the triangle ABC , $DF : DG :: AC : AB$, (by 7. 5.) or $DF : DE :: DG : AB$, therefore DG is equal to DE ; and in the same manner EF will be equal to FG ; and the triangles DEF , DFG , will be equal in every respect, (by 8. 1.) therefore the triangle DFE , will be equiangular to the triangle ABC , as is the triangle DFG .

PROP. VIII.

If the triangle ABC has one angle right, as B , the perpendicular BD shall divide it into two triangles, both similar to ABC ; because they will each have one right angle as the given triangle; and will likewise each have one angle common with it. *Fig. 7.*

PROP. IX.

To divide the line AB into any number of equal parts. Draw the line BC , at pleasure, and with any opening *Fig. 8.*

D

of

of the compass set off so many equal parts upon the line BG, as you intend to divide the line AB into; for example, 4: then from the point G, which determines the last, draw the line GA, to which drawing parallels thro' the several divisions of the line GB, to wit, FH, EL, DM, and the points H, L, M, shall divide the line AB into 4 equal parts.

Demonst. As the line BD: to the line BG: BM: BA, (by 2. 6.) therefore as BD is the $\frac{1}{4}$ of BG, so BM will be the $\frac{1}{4}$ of BA, &c.

To make the work easier, draw AN, parallel to BG, and set off upon AN the same divisions as upon BG; then drawing your lines from the marks on one, to the corresponding marks on the other, such lines will be parallel (by 31. 1.) and the given line will be divided as required.

PROP. X.

To divide the line AB in proportion as the line E is already divided. Draw the parallels AD, and BC, equal to the line E, and lay down the divisions of the line E, upon the line AD, beginning at the point A; do the same on the line CB, beginning at the point C; then drawing thro' the marks HL, the lines HL, they shall divide the line AB, as the lines AD, and BC, (by the preceding) or as the given line E was divided.

PROP. XI.

To find a third proportional to the lines A and B. From the same point E draw two lines at pleasure, making

any angle; then taking EO, equal to the line A, and the lines OH, and EK, equal to the line B, draw HL parallel to OK; so shall the line KL be the third proportional sought.

Demonst. $EO : OH :: EK : KL$, (by 2. 6.) therefore $A : B :: B : KL$.

PROP. XII.

The same must be done, if a fourth proportional to the lines ABC be demanded. For making EO equal to A, OH equal to B, and EK equal to C, KL will be the fourth proportional. *Fig. 10.*

PROP. XIII.

To find a mean proportional between the line and the line B. Upon the same line lay down AB equal to A, and BC equal to B; then having described a semicircle upon AC, draw the perpendicular BD, which will be a mean proportional between AB and BC. *Fig. 11.*

Demonstrat. The angle ADC, being a right angle (by 31. 3.) the perpendicular BD will divide the triangle ADC, into two similar triangles ABD, DBC, (by 8. 6.) therefore $AB : BD :: BD : BC$.

PROP. XIV.

If the equiangular parallelograms A and B are equal, their sides shall be in reciprocal ratio to each other; that is

D 2

the

the side EF is to the side FG, as the side HF to the side FI.

Demonst. Since the parallelograms A, B are equal, they will have the same ratio to a third C (by 7. 5.) but $A : C :: EF : FG$, (by 1. 6.) and $B : C :: HF : FI$; therefore $EF : FG :: HF : FI$.

In the same manner if the sides of the said parallelograms are in reciprocal ratio, we may conclude them equal; because A will be to C, as B to C.

PROP. XVI.

If the four lines A, B, C, D are proportional, a parallelogram E, made with the first and second, and fourth, the two extremes, will be equal to a similar parallelogram F, made with the second and third, the two means, because their sides will (by the preceding) be in reciprocal ratio, that is, the side A : side B :: C : D.

It would be the same thing if the lines B and C were equal; therefore we say, that the square of a mean proportional is equal to the rectangle of the two extremes.

COROL. I

If the lines AB, CD, cut each other within a circle, at the point E, the rectangle contained under the parts AE, EB, of the one will be equal to the rectangle of the parts CE, ED, of the other.

Demonst. The triangles AEC, DEB, have the opposite angles E equal, (by 15. 1.) and

angles C, B resting upon the same arch AD are also equal, (by 21. 3.) therefore they are similar, and consequently (by 4. 6.) $AE : ED :: CE : EB$; therefore the rectangle contained under AE, EB, will be equal to the rectangle contain'd under DE, CE (by the preceding.)

COROL. II.

If from the point A without a circle you draw the line AC cutting the circle, and the line AD a tangent to it; the rectangle *Fig. 15.* contain'd under the line AC, and the part AF, without the circle, will be equal to the square of the line AD.

Demonst. The triangles CAD, ADF, are similar, having the angle A common, and the angle C equal to the angle ADF, (by 32. 3.) therefore $AC : AD :: AD : AF$, (by 4. 6.) and consequently the square of the mean proportional AD will be equal to the rectangle of the two extremes AC, AF, (by the preceding.)

PROP. XIX.

If the triangles A and B are similar, the triangle A will be to the triangle B in a duplicate ratio to the ratio between their homologous sides. That is to say, if the base LM is double the base GH, the triangle A will be quadruple the triangle B. *Fig. 16.*

Demonst. Dividing equally the lines LM in the point D, and LC in the point E, and drawing DE, DC, it will appear, that the triangle ELD is equal to the triangle B, (by 4. 1.) because

cause the angle L is equal to the angle G , and the sides LD , LE , equal to the sides GH and GF . But the triangle LED , is $\frac{1}{4}$ of the triangle LCM , being but $\frac{1}{2}$ of the triangle LCD , (by 1. 6.) which is itself but $\frac{1}{2}$ of the triangle LCM ; therefore the triangle B is $\frac{1}{4}$ of the triangle LCM .

In the same manner it may be demonstrated, that if LM is triple GH , or quadruple, the triangle A would accordingly be nine, or sixteen times the triangle B , &c.

PROP. XX.

Similar polygons A and B are all in a duplicate ratio of their homologous sides, because they are composed of similar triangles.

PROP. XXIII.

If the parallelograms A and B are equiangular, the ratio of the parallelogram A to the parallelogram B will be composed of the ratio of the line CD , to the line DE , and of the ratio of the line GD , to the line DF .

Demonst. If CD is $\frac{2}{3}$ of DE , the parallelogram A will be $\frac{4}{9}$ of the parallelogram H , (by 1. 6.) and if GD is $\frac{1}{2}$ of DF , the parallelogram H will be $\frac{1}{4}$ of the parallelogram B , and the parallelogram A will be $\frac{1}{9}$ of $\frac{1}{4}$, or $\frac{1}{36}$, i. e. $\frac{1}{9}$ of the parallelogram B .

PROP.

PROP. XXX.

If the line AB be divided in the point C, so that the rectangle contained under AB, and BC, be equal to the square of AC, *Fig. 19.* (by 11. 2.) then is such line divided in extreme and mean proportion; because $AB : AC :: AC : CB$, (by 16. 6.)



BOOK XI.

DEFINITIONS.

1. **A** *Solid* is a magnitude wherein we consider length, breadth, and thickness.
2. A line is said to be right upon a plane, when it makes right angles with all the lines of the plane. Thus the line *AB* will be right to the plane *CDEF*, if the angles *CBA*, *DBA*, *EBA*, &c. are right. From this definition it follows, that two lines cannot be perpendicular to the same plane, and fall upon the same point.
3. If the planes *G* and *D* cut each other, some of their points, as *AB*, will be found in both planes, and a right line drawn thro' those points will also pass thro' both planes, and is therefore call'd their *common Section*.
4. The angle made by the plane *A*, with the plane *B*, is the same as would be made by the lines of both, which are perpendicular to their common section, as *DGC*, *CEF*.

5. Solids

5. Solids are equal in every respect, when they are bounded with an equal number of equal surfaces, and disposed in the same manner.

6. Similar Solids are bounded with an equal number of similar surfaces, and disposed in the same manner.

7. A solid angle is form'd by the concurrence or meeting of more than two planes in the same point, not making one plane, as B. Fig. 4.

8. Parallel planes are such as being infinitely prolong'd will not meet.

9. A *Prism* is a body bounded with two right-lin'd parallel planes, equal in every respect, and by an infinite number of right lines passing from the one to the other, as the body A. Fig. 5.

10. *Parallelepipedis* are bodies bounded by six planes, of which every opposite two are parallels, as B. When these six planes are squares, the parallelepipedon is a *Cube*, as C. Fig. 6.

11. A *Pyramid* is a body bounded by one right-lin'd plane, and an infinite number of right lines, draw from the circumference of the said right-lin'd plane, to a point without it, as D. Fig. 7.

D 5

12. A

12. A *Cylinder* is a body bounded by two circular planes, equal, and parallel, and an infinite number of right lines passing from the one to the other, as E. The line connecting the centers of the two circles is call'd the *Axis*, as EF.

13. A *Cone* is a body bounded by one circular plane, and an infinite number of right lines, drawn from the circumference, to a point without its plane, as H. The line joining the center and the said point, at the top, or vertex of the *Cone*, is call'd its *Axis*, as EH.

14. A *Globe*, or *Sphere*, is a body whose extreme parts are all equally distant from a certain point within it, which point is call'd its center, as P.

P R O P. VI.

If the lines AB, CD, are perpendicular to the same plane, they are parallel.

Demonst. They will be in the same plane, to wit, that which would be described by the line CA, if remaining perpendicular upon the plane, they both meet the line CA : farther, they will be perpendicular to the line AC, (by *Def. 1.*) therefore they will be parallel.

P R O P.

PROP. VIII.

If the lines AB , CD , are parallel, and AB right angled upon a plane, CD shall be perpendicular upon the same plane; *Fig. 11.* otherwise, one might imagine the line CE right upon the plane, which would then be also parallel to the line AB , (by the preceding) which cannot be.

PROP. IX.

If the lines A and B are parallel to a third C , they will be parallel to one another; for imagining a plane, upon which the *Fig. 12.* line C is right or perpendicular, the lines A and B , which are parallel to the line C , will be right upon the same plane, and consequently parallel to one another.

PROP. X.

If the line AB is parallel to the line CD , and the line BE parallel to the *Fig. 13.* line DF , the angle ABE will be equal to the angle CDF .

Demonst. The lines AB , CD , and EB , DF , being equal, the lines AC , EF , will be equal, and parallel to a third BD , and consequently to one another; therefore the lines AE , CF , will be also equal, (by 33. 1.) and all the sides of the triangle ABE , will be equal to those of the triangle CDF , and consequently (by 8. 1.) the angle ABE equal to the angle CDF .

PROP.

P R O P. XVI.

If the parallel planes *A* and *B* are cut by a third plane *C D F E*, the common sections *CD*, *EF*, will be also parallel; because they will be in the same plane, and can never meet, any more than the two planes *A* and *B*.

P R O P. XVIII.

If the line *AB* is right upon the plane *O A F D*, all the planes which can pass thro' the line *AB* will be perpendicular to the same plane; because the line *AB*, which determines the angle every such plane will make with the plane *O A F D*, stands itself at right angles to the same plane, (by *Def. 2.*)

P R O P. XXIV.

If a body is bounded with six planes, of which every opposite two are parallel, as *A* and *B*, *C* and *D*, *E* and *F*, those opposite planes are equal, and similar parallelograms one to another.

Demonst. Since the parallel planes *A* and *B* are cut by the plane *D*, the common sections *GH*, and *KI*, will be parallel, (by 16. 11.) so will also *IH*, *KG*, and the figure *HIKG* will be a parallelogram. In the same manner the figures *G K L N*, *K L M I*, *I M O H*, *O H G N*, *O N L M*, will be parallelograms; and consequently the lines answering each other are equal, as *GK*, *NL*; *OM*, *HI*; and *GH*, *ON*; *ML*, *KI*.

KI. Farther, the angles which answer each other are equal, as the angle HGK to the angle ONL , because the lines HG , ON , GK , NL , are parallel: Therefore the planes D and C , and the other correspondent ones, are equal, and similar parallelograms.

PROP. XXVIII.

If the parallelepipedon A be cut by the plane BC , passing thro' the diagonals of two opposite planes, the two parts will be- *Fig. 17.*
come two equal prisms, since they will be bounded with an equal number of equal and similar planes, disposed in the same manner.

PROP. XXIX.

If the parallelepipeds $ABEFDHG$ and $ABNMDOL$ have the same base ABD , and the same height DH , *Fig. 18.* they are equal; because they have the prism $ABDHGNM$ common, and the prism $ABNMEF$ is equal to the prism $DGHOL$, therefore (by *Ax. 3.*) the two parallelepipeds are equal.

PROP. XXXIII.

If the parallelepipedon A is similar to the parallelepipedon E , and the side BC be double the side FG , which is homologous to it, the parallelepipedon A will be eight times the magnitude of the parallelepipedon E .

Demonst.

Demonst. Since BC is double FG, BD will be also double FH, and DL double HI, If then you divide the parallelepipedon A, in its length, breadth, and depth, into two equal parts, you will make eight parallelepipeds, any one of which, as M, having all its planes equal, and similar to the planes of the parallelepipedon E, will consequently be equal to it, in every respect.

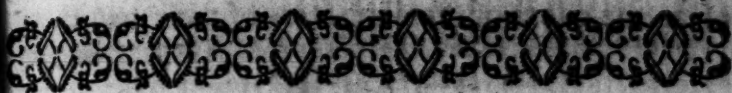
In the same manner if the line BC be triple the line FG, it might be proved, that the parallelepipedon A would contain twenty-seven times the parallelepipedon E; from whence it follows, that similar parallelepipeds are in triplicate ratio, of their homologous sides; as are also similar prisms, which are the halves of similar parallelepipeds.

PROP. XXXVIII.

If the plane A is right or perpendicular upon the plane B, and from the point A be drawn
Fig. 20. a line AB, perpendicular to their common section CD, it will be right upon the plane B.

Demonst. If from every point of the line CD were drawn lines all right to the plane B, they would constitute the plane A, and one of those lines must be the line AB.





B O O K XII.

P R O P. I.

IF the Polygons R and O, inscribed in circles, are similar, the side AE will be to the side CF as the diameter AB to the diameter CD; because the triangles ABE, CDF, are similar. *Fig. 1.*

The same may be demonstrated of all the lines which bound a similar arch.

P R O P. II.

The greater the number of sides to a polygon, whether inscrib'd in a circle or circum-scrib'd about it, the less it differs from a circle; therefore a polygon having a very great number of sides, scarce differs at all from a circle: from whence it follows, that as polygons are in a duplicate ratio of the radii of the circles in which they are inscribed, (by the preceding) circles also are in a duplicate ratio of their radii. *Fig. 2.*

C O R O L L A R Y.

Cylinders may be also taken for prisms; from hence it may be concluded, that similar Cylinders are

are in a triplicate ratio of their homologous sides,
or as the cubes of the diameters of their bases,

P R O P. III.

Two pyramids A, and B, having
Fig. 3. the same base DCE, and being between
the same parallels, DE, AB shall be equal.

Demonst. The triangle DCE is composed of
an infinite number of lines parallel to the line DE.
If from the ends of these lines other lines be drawn
to the point A, they will constitute an infinite
number of triangles, composing the pyramid A.
In like manner the pyramid B would be composed,
if the said lines be brought to the point B.
But every one of those triangles which compose
the pyramid A will be equal to each of those which
form the pyramid B, (by 37. 1.) therefore the
whole pyramid A shall be equal to the whole
pyramid B.

P R O P. VII.

The triangular pyramid DEFC is but the
third part of the prism ABCDEF.
Fig. 4. having the same base, and equal height
because the whole prism contains two
more pyramids, viz. ABFC, and ADFC, which
having equal bases ABF, and ADF, and the same
vertex C, are equal, (by the preceding) and one
of them, viz. ABFC, is equal to the pyramid
DEFC, (by the preceding) because it has both
the same base, and the same height as the prism.

It is the same thing with pyramids which are not triangular, because they may be divided into triangular pyramids. From whence it follows, that similar pyramids are in a triplicate ratio of their homologous sides ; and consequently cones (which may be taken for pyramids of an infinite number of sides) as they also are a third part of cylinders, of the same base and heighth, are likewise in a triplicate ratio of their homologous sides.

P R O P. the Last.

A Globe may be considered as composed of an infinite number of pyramids, whose vertices meet at the center of the Globe, *Fig. 5.* their bases composing its surface, and their sides forming its radii. So that each pyramid composing the globe A, being to each pyramid composing the globe B, in a triplicate ratio of the radius AC, to the radius BD ; the globe A will be to the globe B in a triplicate ratio of the radius AC, to the radius BD.

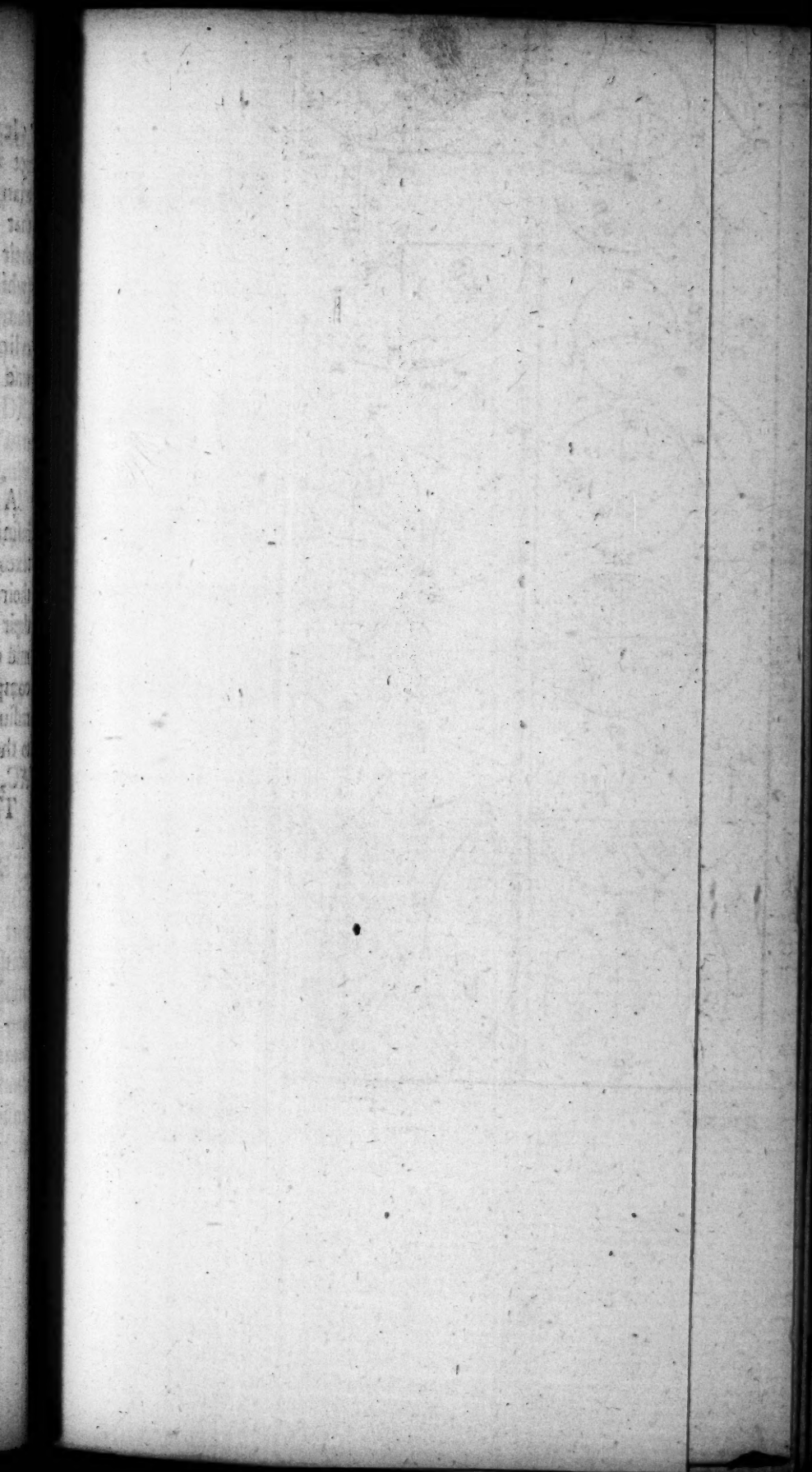
The same may be said of all other similar bodies,

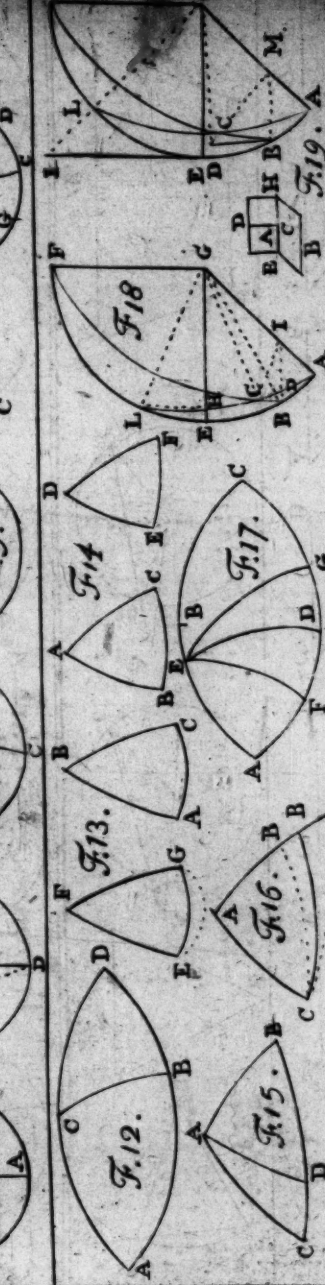
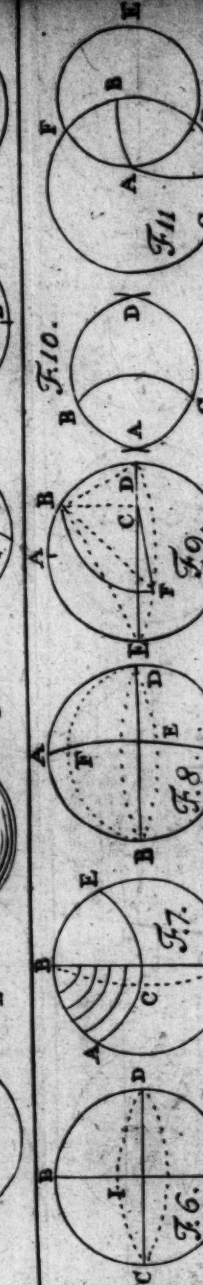
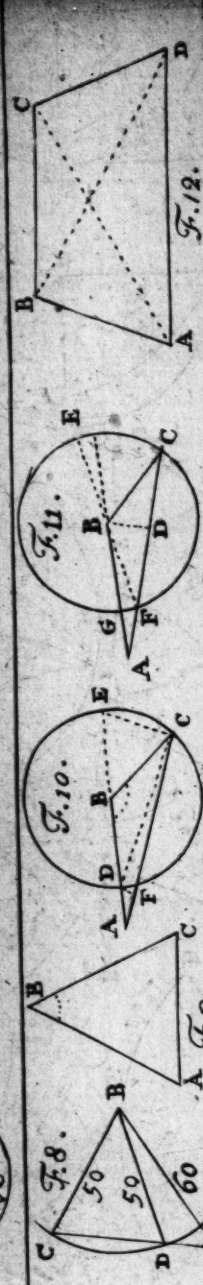
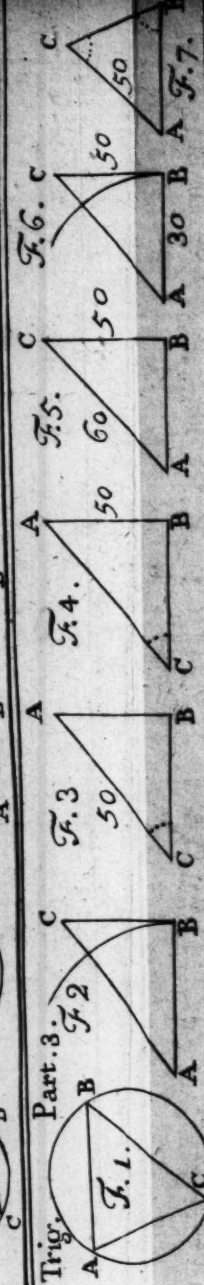
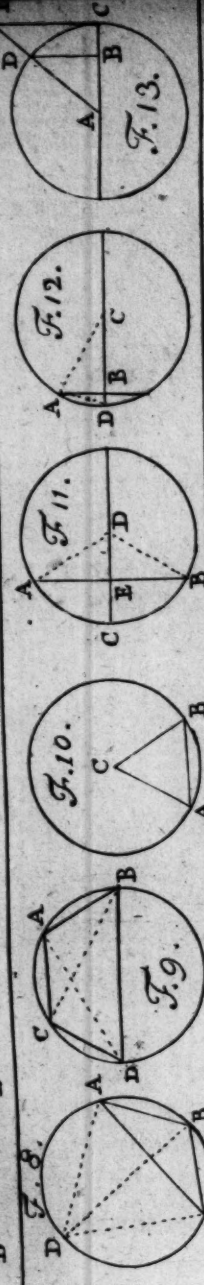
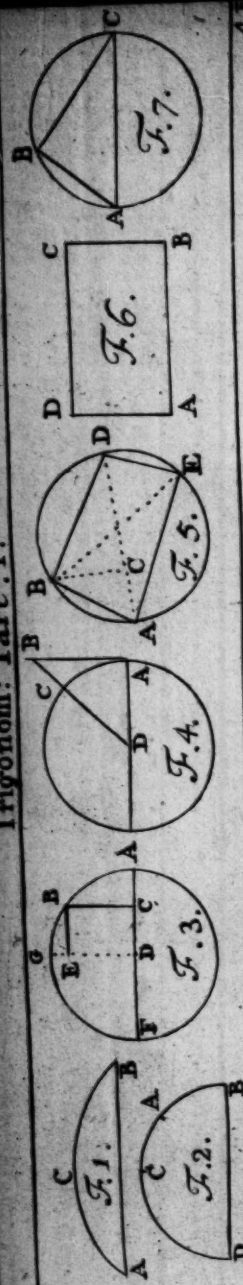


The same may be said of all other fluid bodies.



4019





TRIGONOMETRY.

TRIGONOMETRY is a science by which we learn the resolution of triangles; that is, to find the angles and sides unknown, by the angles and sides known.

PART I.

The Construction of the Tables of Sines, Tangents, and Secants.

DEFINITIONS.

AN Arch is a part of the circumference of a circle, as ABC; *Fig. 1.* The right line AB extending from the one end of the arch to the other, is call'd the chord, or subtendent of the arch.

The whole circumference of every circle is supposed to be divided into 360 degrees, each degree into 60 minutes, and each minute into 60 seconds, &c.

2. What

2. What an arch wants of 90 degrees, is call'd its *complement*; and its defect of 180 degrees, is call'd its *supplement*. Thus the arch AC is the complement of the arch AB, if BAC contains 90 degrees; and AD is its supplement, if the whole figure is a semicircle.

3. The *right line* of an arch is a right line falling from the one end of the arch perpendicularly on the diameter that terminates in the other end. Thus BC is the right line of the arch BA, or of its supplement BF.

4. The *sine complement*, or *co-sine* of an arch, is the right line of its complement. Thus BE, which is the right line of BG, the complement of BA, is call'd the sine complement, or co-sine of BA; and since BE is always equal to DC, therefore DC is call'd the sine complement of the arch BA. *V. Fig. 3.*

5. The *versed sine* of an arch is that part of the diameter contained between the arch and its right line. Thus CA is the versed sine of the arch BA, and CF of the arch BF. *V. Fig. 3.*

6. The *radius* of a circle, as GD, is call'd the *sine total*, because it is the greatest right line. *V. Fig. 3.*

7. The *tangent* of an arch is a right line touching one end of the arch, and drawn without the arch, parallel to the right line till it meets the radius produced thro' the

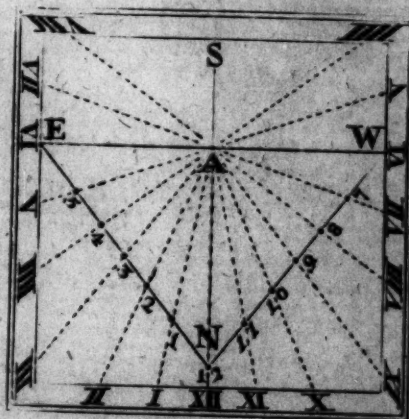
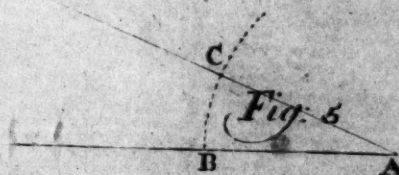
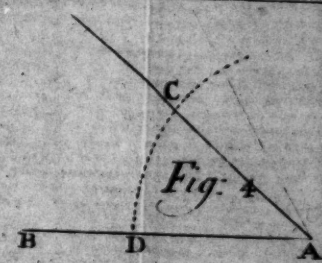
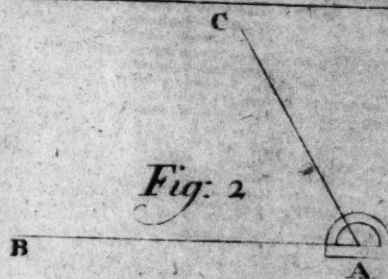
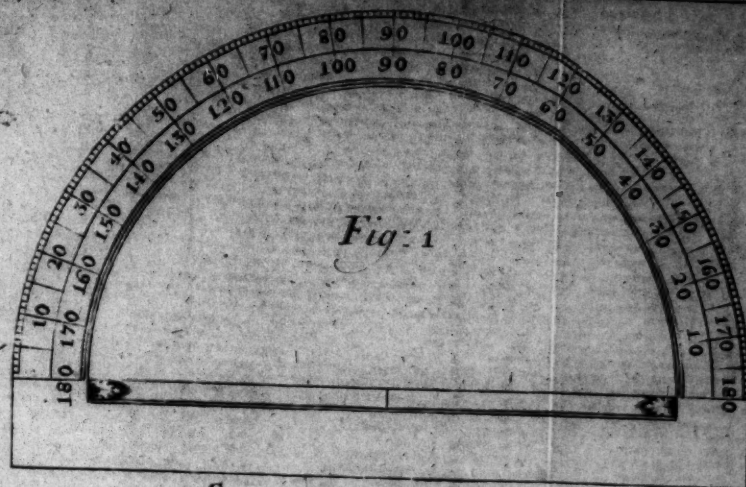
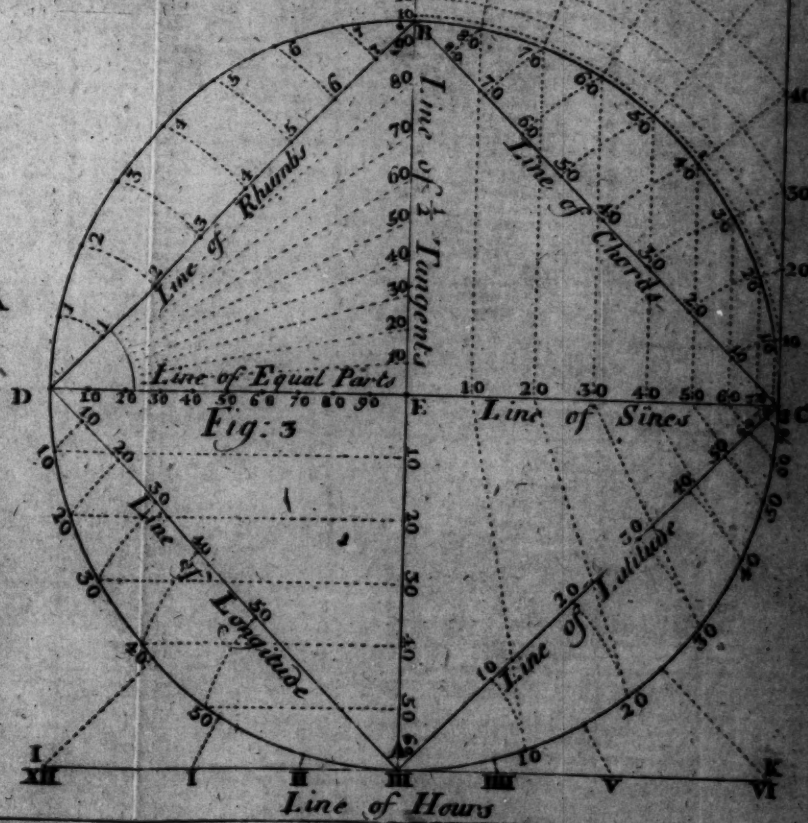


Fig: 6



Line of Seconds

Line of Tangents

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50

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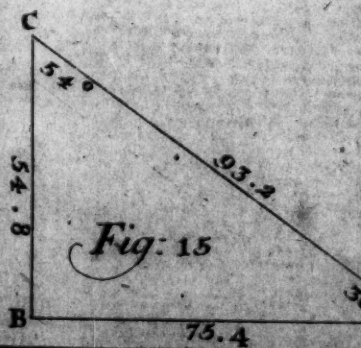
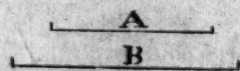
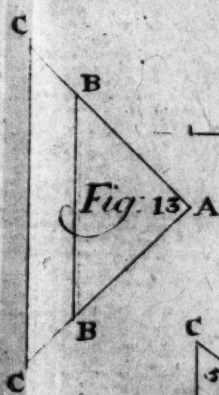
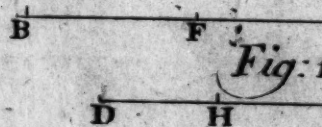
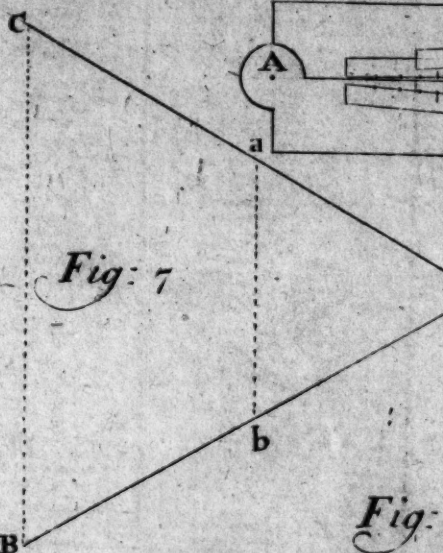
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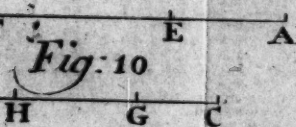
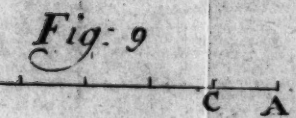
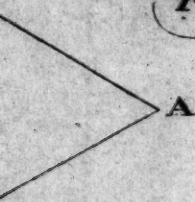
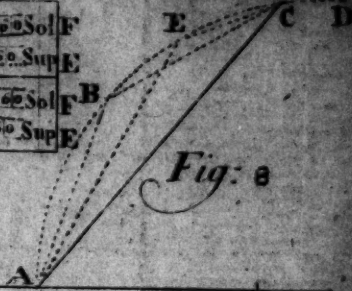
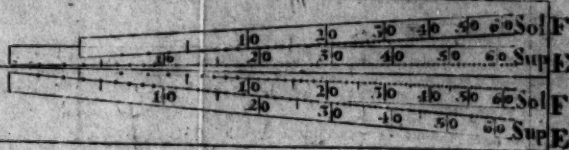
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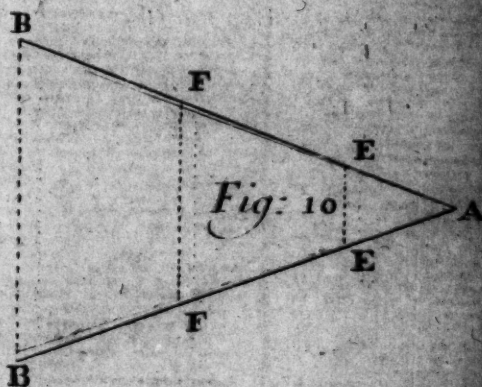
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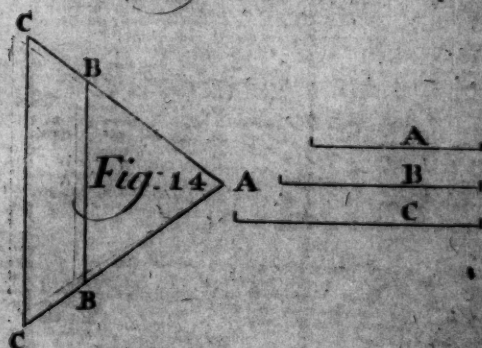
A



A 3

B 7

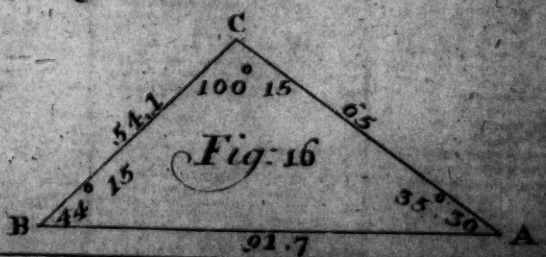
Fig: 12



A

B

C



36°

100° 15'

Fig: 16

91.7

the other end: The radius so produced is call'd a Secant. Thus AB is the tangent of the arch AC, and DB the Secant.

Thus the Tables of Sines, Tangents, and Secants, being valued by the proportion they bear to the radius of every circle, and that proportion being in numbers often irrational, the Tables cannot be perfectly exact; but that there may be no sensible error in their application to practice, the radius of each circle is divided into 100000 equal parts; and the Tables shew how many of such parts are contain'd in every Sine, Tangent, and Secant, neglecting the fractions.

L E M M A. I.

In every four-sided figure ABDE, inscribed in a circle, the rectangle made by the diagonals AD, EB, is equal to the two rectangles made by the opposite sides of the said figure, to wit, the rectangle made by EA, DB, and the rectangle made by ED, AB. Draw AC, so that the angle ABC may be equal to the angle DBE. Fig. 5.

Demonstrat. The triangle ABC is similar to the triangle EDB; for besides the equal angles ABC, DBE, the angle BED is equal to the angle BAD, (by 21. 3.) Therefore (by 4. 6.) $AB : AC :: EB : ED$, and consequently the rectangle of the extremes AB, ED, will be equal to the rectangle of the means AC, EB, (by 16. 6.) In the same manner the rectangle upon AE, BD, will be equal to the rectangle upon DC, EB; because the triangles BCD, BAE, are similar, and conse-

consequently the rectangle upon EB, and the whole line AD, will be equal to the two rectangles upon AB, ED, and upon AE, BD.

LEMMA II.

If the area of the rectangle ABCD, and one of its sides AB, be given, the other side AD, may be found by dividing the rectangle, by the side known.

LEMMA III.

If $A : B :: C : D$, and A, B, C given, D may be found, by multiplying B by C, and dividing the product by A, since that product is equal to the product of A by D, (by 16. 6.)

PROP. I.

The diameter AC and chord of an arch AB, being given, the supplemental BC may be also found.

Demonst. The angle B being right, the square of the diameter AC will be equal to the square of the line AB, and the square of the line BC, (by 47. 1.) and consequently if the square of the line AB be subtracted from the square of the diameter AC, the square of the line BC will remain; the root of which being extracted, will give the line BC.

PROP. II.

The chord BC of an arch being given, the chord CA of an arch double CB may be found. Draw the diameter BD, and the lines DA, DC, then will DA be equal to DC. *Fig. 8.*

Demonst. DA, DC, may be found by the foregoing. Multiply AB by CD, and CB by AD, the sum of the two products will be equal to the rectangle made upon BD and AC, (by *Lemma 1.*) therefore dividing by BD, you have CA, (by *Lemma 2.*)

PROP. III.

The chord AB being known, the chord DB of an arch triple to AB may be found. Draw AC, and CD, equal to AB; Draw also AD, CB, which may be known as before. *Fig. 9.*

Demonst. Multiply the chord AD by the chord AC, and the product will be equal to the two rectangles upon DC, AB, and upon CA, DB, (by *Lemma 1.*) then subtracting the rectangle upon CD, AB, the remainder will be the rectangle upon CA, DB, which being divided by CA, gives DB.

In the same manner may be found the chord of an arch quadruple, quintuple, sextuple, &c.

PROP. IV.

The chord of an arch of 60 degrees B is equal to the radius CA. *Fig. 10.*

Demonst.

Demonst. The angle C contains 60 degrees, the two others A, B, contain what remains to make up the two right angles, that is, 120 degrees, (by 16. 1.) and because they are equal, each must contain 60 degrees, (by 5. 1.) as the angle C and consequently the line AB will be equal to the lines CA, or CB, (by 6. 1.)

P R O P. V.

Fig. 11. Half the chord AB, is the right sine of half its arch ACB.

Demonst. If the line AB be divided equally in the point E, the triangles AED, BED, will be equal in every respect, (by 8. 1.) and consequently AE will be the right sine of AC, (Def. 3.) which is the half of ACB, since the angles ADC, BDC, are equal.

C O R O L L A R Y.

The chord of 60 degrees being equal to the radius, contains 100000 parts, therefore the right sine of 30 degrees, being half the cord of 60 degrees, will contain 50000.

P R O P. VI.

Fig. 12. If the right sine AB, of an arch AD, be known, the chord AD may be also known.

Demonst. Subtract the square of the sine AB from the square of the radius AC, the remainder will be the square of the line BC, (by 47. 1.) whose square root being subtracted from the radius CD, will give the versed sine BD; add therefore

the square of BD, to the square of BA, and the sum will be the square of the chord AD (by 47. 1.)

COROLL. I.

Having the chord AD, the half thereof will be the right fine of 15 degrees, the chord of which may be found by the foregoing, and its half will again be the right fine of 7 degrees 30 minutes. Thus descending by continual halving, you will find that the right fine of $52''. 44'''. 3''''. 45'''''$. is 2554. You must observe, at the same time, that the fine of $1'. 45'''. 28'''. 7'''. 30''''$. is double the fine of $52''. 44'''. 3'''. 45'''''$. so that the fine is to the fine as the arch to the arch; therefore without danger of mistake, you may say, as $52''. 44'''. 3'''. 45'''''$. are to $60''$. so 2554 to the fine of $60'$. which will consequently be 2909.

COROLL. II.

Having thus found the right fine of 1 minute, you must double it to get the chord of 2 minutes, which by *Prop. 2* will give the chord of an arch double, or the chord of $4'$. of which the half will be the fine of $2'$. and by *Prop. 3*. it will give you the chord of a triple arch, or of $6'$. the half of which will be the fine of $3'$. Lastly, this same chord of $2'$. will give the chord of an arch quadruple, quintuple, sextuple, &c. or of $8'$. of $10'$. of $12'$. &c. the half of which will be the fine of $4'$. of $6'$. &c.

E

PROP.

PROP. VII.

Having found the right sines of all the arches, you will have their tangents, and their
Fig. 13. secants. For example, you may have the tangent CE, if you know the sine DB, and the sine complement AB.

Demonst. The triangles ABD, ACE, are rectangled, and have the angle A common; therefore they are similar, and consequently the sine complement AB, will be to the right sine BD, as the radius AC to the tangent CE, (by 4. 6.)

In the same manner for the secants, the sine complement AB will be to the radius AC, as the radius AD to the secant AE, (by 4. 6.)

Note, To the end that the fractions neglected in the beginning, may not cause any great error in the progress, it will be proper to take them into the produce, so as to be constantly within the half unit, in the lowest place.





PART II.

LOGARITHMS.

LOGARITHMS are a set of artificial Numbers, by means of which the operations of Multiplication, and division, are performed by Addition and Subtraction ; and the extraction of the Root of any power by a single division of the Logarithm of the power by the index or exponent of such power ; thus dividing it by 2, the quotient is the Logarithm of the square root, and by 3, that of the cube root, &c.

DEFINITIONS.

1. Arithmetical Progression, is a succession of numbers equally exceeding each other, as 0, 1, 2, 3, 4, 5, &c. or 1, 3, 5, 7, 9, &c.
2. Geometrical progression, is a succession of numbers, of which the first is to the second, as the second to the third, and as the third to the fourth, &c. as 1, 2, 4, 8, or 1, 3, 9, 27.
3. If exactly under the geometrical progression A, beginning with 1, you place the arithmetical progression B, beginning with 0, the numbers of

E 2
the

the arithmetical progression will be the logarithm of the corresponding numbers in the geometrical progression. For example, the third number of the arithmetical, will be the logarithm of the third number of the geometrical; that is, the number C is the logarithm of the number D, so is E of F and G of H.

$$\begin{array}{ccc} & D F & H \\ A \div & 1, 2, 4, 8, 16, 32, 64. \\ B \div & 0, 1, 2, 3, 4, 5, 6. \\ & C, E, & G. \end{array}$$

LEMMA I.

In every geometrical progression A, the numbers immediately following each other, have the same common ratio between them; but those which are separated by the interposition of one term or place, as B, C, have a ratio, double to the general one; and those which are separated by the interposition of two places, as B, D, have a triplicate ratio, &c. (by *Lem.* I. 6.)

$$\begin{array}{ccc} A \div & 1, 2, 4, 8, 16, 32. \\ & B, & C, D. \end{array}$$

COROLL. III.

From hence it follows, that if two numbers in a geometrical progression are at the same distance from each other, as two other numbers, they will have the same ratio; and if they have the same ratio, they are at the same distance.

LEMMA

L E M M A II.

In every geometrical progression A, whose first term is unity, if the number B be multiplied by the number C, the product shall be a number in the same progression, as far distant from the number C, as the number B is from unity, to wit, D.

$$A \div 1, 2, 4, 8, 16, 32, 64, 128.$$

B,
C
D.

Demonst. If multiplying B by C produces D, must be $1 : B :: C : D$, therefore (by *Lemma preceding*) B will be as far distant from unity, as from C.

On the contrary, if D be divided by B, the quotient will be C, as far distant from D, as the divisor B from unity; because in every division, the dividend is to the quotient, as the divisor is to unity.

L E M M A III.

In every arithmetical progression commencing with 0, if the number marked C, be added to the number D, the sum will be E; the same distance from D, as C from 0,

$$B \div 0, 1, 2, 3, 4, 5, 6, 7.$$

C,
D,
E.

Demonst. C will be more or less distant from 0, according as it contains more or less times the common difference of the progression. In the same manner E will be more or less distant from D, according as it contains more or less times the same common difference above D. But since E is the

E 3
sum

sum of C and D, it will contain the same number of times the common difference more than D, as C does more than 0; therefore E will always be as far distant from D, as C from 0.

In the same manner, it may be proved, that if, on the contrary, the number C be subtracted from the number E, the remainder will be the number D, as far distant from E, as C from 0, since E contains as many times the common difference above D, as C does above 0.

P R O P. VIII.

If in the arithmetical progression B, answering the geometrical progression A, the number C, the logarithm of the number D, be added to the number E, the logarithm of the number F; the sum G will be the logarithm of the number H, and the number H will be the product of the number D multiplied by the number F.

	D	F	H
A $\div$	1, 2, 4, 8, 16, 32, 64.		
B $\div$	0, 1, 2, 3, 4, 5, 6.		
	C	E	G

Demonst. Since in the arithmetical progression the number C immediately follows 0, if to it be added the number E, the sum G must immediately follow the number E, (by *Lem. 3.*) so in the geometrical progression, since D immediately follows 1, if it be multiplied by F, the product shall immediately follow F, (by *Lem. 2.*) Therefore since E answers to F, so shall G to H, and will consequently be its logarithm.

In the same manner it may be demonstrated that if C, the log. of D, is subtracted from G, the

log. of H, the remainder E, will be the log. of the number F, and the number F, will be the quotient of H, divided by D.

C O R O L L A R Y.

The Use of Logarithms.

1. If you are to find a fourth proportional to three numbers given A, B, C, having found in some Table of logarithms, the logarithms of the given numbers, add the log. of B and C, and from their sum E, subtract the log. of the number A, and the remainder D, shall be the log. of the fourth number required F.

A	1097	—	3,0402066
B	1078	—	3,0326188
C	964	—	2,9840770

E	—	6,0166958
A	—	3,0402066

D	—	2,9764892
---	---	-----------

F 947 $\frac{1}{8}$

Demonstrat. To find a fourth proportional, the rule is, to multiply the second and third, and divide the product by the first; but in subtracting the log. of the number A, from the sum of the logs. of the numbers B and C, we find the log. of the product of the numbers B and C, divided by the number A; therefore that log. must be the logarithm answering to the fourth proportional sought.

2. To find the square root of any number, as 801, having from any table found its logarithm, which

E 4

which is 3,9912704, the half thereof, viz. 1,9956352, will be the log. of the root sought, to wit, 99. For adding the log. of 99 to itself, it gives the product of 99 multiplied into itself, that is its square.

In the same manner taking the third part of the logarithm of any number, you have the log. of its cube root; and taking the $\frac{1}{4}$ you have the log. of its biquadrate root, &c..

PROP. IX.

The Construction of Logarithms correspondent to the natural Numbers.

1. We begin with the geometrical progression A, in which the terms increase by the ratio of one to ten: for the logarithms of which we set down the arithmetical progression B, in which the terms exceed each other by 1000000. The sequel will make it clear why we begin in this manner.

A	B	
1	= 0,000000	A 1,0000000 — 0,000000
10	— 1,000000	D 3,1622777 — 0,500000
100	— 2,000000	B 9,0000000
1000	— 3,000000	
10000	— 4,000000	C 10,0000000 — 1,000000
100000	— 5,000000	

2. We seek the logarithms of the numbers between 1 and 10, beginning with finding the log. of 9, thus:

Add to each of the three numbers 1, 9, 10, seven cyphers, to make the numbers A, B, C, which have the same ratio as 1, 9, and 10, and consequently must have the same logarithm; therefore multiplying the number A, by the number C, and extract-

extracting the square root of the product, will produce the number D, a mean proportional between A and C, (by 16. 6.) and consequently to have its log. half the difference between the log. of the number A, and that of the number C, must be taken, *viz.* 500000, which being added to the log. of the number A, will make 0,500000, the log. of the number D. In the same manner may be found the logs. of all numbers, which will be mean proportionals between two others, of which the logs. are already known.

Thus multiplying the number D by the number C, and extracting the root of the product, you will find the mean proportional between D and C, and its log. will be 0,750000. The same work is continued in searching the mean proportionals between two numbers still nearer to B, till at last the number B becomes itself a mean proportional between two numbers whose logarithms are known, and consequently its own logarithm will be known, to wit, 0,954242, which will be also the logarithm of the number 9.

3. In the same manner we find the logarithm of 7, and of 2. Then half the logarithm of 9, gives the log. of its square root 3; the double of the log. of 2, will give the log. of its square 4, subtracting the log. of 2, from the log. of 10, gives the log. of 5, adding the log. of 2, to the log. of 3, you have the log. of 6; and adding the log. of 2, to that of 4, you will find the log. of 8.

4. In the same manner we find the logarithms answering to the numbers between 10 and 100, by making use of the logs. of numbers already known, to discover those of their products and

E 5

roots

roots; and for the rest, we do as for the numbers 9, 7, and 2.

Note 1. The Tables of Logarithms are usually carried on to 10000.

2. When we seek mean proportionals, we often meet with numbers which have no exact root, and yet we neglect the fractions; so that those logarithms are not perfectly exact; but when the root is extracted to many places, such remainders are very inconsiderable, so there can arise no sensible error in their application.
3. The logarithms of all the numbers between 1 and 10, begin with 0; of those between 10 and 100, with 1; of those between 100 and 1000, with 2, &c. This first figure of a logarithm is usually separated from those following by a point annex'd, and is call'd the Index, or Characteristic, because it shews how many places the corresponding number (when integral) consists of, which is always one more than the units in the Index.

Note, The logarithm of a fraction is the difference of the logarithms of both terms; therefore subtract the logarithm of the denominator from the logarithm of the numerator, and the remainder is the logarithm of the fraction, and the number found in the Table corresponding thereto, is the decimal fraction of the same value as the vulgar fraction given; but the index of the logarithm of a proper fraction is negative, and has a short line—for minus put over it, and tells the place that the first significant figure of the number is below the unit's place; so that the index of a logarithm, whether negative or affirmative, is only to shew the place the first significant of the number possesses from the unit's place, either below or above it, and is no part of the logarithm to be sought for in the tables, where we use only the fraction, or part to the right hand of the point, which is always affirmative, as Mr. Gardiner in page 1. of the Explanation to his Tables makes evident.

PROP. X.

The Construction of Logarithms for the Sines.

1. The sine total has for its log. 100000000, or perhaps 100000000000; but in most tables the last three figures are neglected.

2. The natural sine of 89 degrees 59'. is found to be 9999999... to find its log. we seek in the ordinary tables the log. of 9999. which is 3,9999566, to which we add the log. of 1000000, which is 6,0000000, and we have 9,9999566, the log. of 9999000... Then subtracting 9999000... from the sine total 100000000... the remainder will be found 1000... In the same manner subtracting the log. 9,9999566 from the log. 10,0000000, the remainder will be 434, by which we may observe, that 1000... the difference of the numbers, gives 434 for the difference of the logarithms. Afterwards we seek the difference between the given sine, and the sine total; which finding to be 1... we say, if 1000... gives 434, then 1... will give $\frac{434}{1000}::$ which being subtracted from the log. of the sine total, will give 99999999 $\frac{566}{1000}::$ for the log. of the given sine.

A	10000000	0000000
B	9999999	0000000
C	I	0000000
D		9999999
E	9999998	0000001
F		9999998
G	9999997	0000003
H		9999997
I	9999996	0000006

3. In the same manner may be found all the logarithms of the sines; but to do it more easily, we form a geometrical progression, of which 10000000 and

and 9999999 are the two first terms ; after this manner we add seven cyphers to the fine total, to make the number A, and the same to the fine 9999999, for the number B, the difference of these two numbers will be C, 1,0000000, or $\frac{1}{10000000}$ of the number A ; then subtracting seven cyphers from the number B, it gives the number D, which will also be $\frac{1}{10000000}$ of the number B ; consequently if you subtract it from the number B, you will have the number E, which will have the same ratio to the number B, as the number B to the number A ; for as E will be $\frac{9999999}{10000000}$ of the number B, so B will be $\frac{9999999}{10000000}$ of the number A. The same must be done for the number G, I, &c. to 100 terms, assigning them logarithms, which shall diminish by the same difference, as is between the log. of the first number, and the log. of the second, viz. $\frac{434}{10000}$, and you will find among those numbers, 15 fines, with their logarithms.

Further, you must raise by the same way a progression of 50 terms, of which the first must be the fine total 10000000, and the second 9999900, the last term of the foregoing progression, and you will find several other fines with their logarithms.

Lastly, you must make several other progressions, having the fine total for their first term, and the last term of the preceding progression for their second, of which the last figures will be cyphers.

N. B. You must remember to end the progressions when their terms will not increase enough to reach the fine which follows. By this means you may find the log. of most of the fines. For the others, they must be found after the manner of that of 9999999.

P R O P.

PROP. XI.

The Construction of the Logarithms of the Tangents, and Secants.

1. Having found the logarithms of all the sines, those of the tangents may be easily discovered, since the sine complement is to the right sine, as the sine total to the tangent, (by *Trig. Prop. 7.*) for if the log. of the sine total be added to the log. of the right sine, and from the sum the log. of the sine complement be subtracted, the remainder will be the log. of the tangent.

2. For the same reason (as above) if from double the log. of the sine total, the log. of the sine complement be subtracted, there will remain the log. of the secant.

Note 1. The log. of a fraction is the difference between the logarithms of the numerator and denominator.

2. If in your operations you meet with a log. not to be found in the tables, look out for one the nearest you can find to it, without regarding the index, or characteristic; then adding as many cyphers to the number answering to it, as there are units in the index of the log. given, more than in the index of that found, and you will have the number of the log. given to a small fraction, which may be also found by this analogy; if the difference between the log. found, and that which follows it in the tables gives 1, the difference between the log. given and that found (without regard to the index) shall give the fraction, which, as the log. given is greater or less than that found, must be accordingly added, or subtracted, without regarding the index, or characteristic: The Demonstration of which is easy.

PART



P A R T III.

PLAIN TRIGONOMETRY: *Or The*
Resolution of right-lin'd Triangles.

S U P P O S I T I O N S.

1. **A**LL triangles are resolved by the rule of analogy, or proportion, in which three known numbers (except the three angles) are given; for multiplying the two last, and dividing the product by the first, the quotient gives the fourth proportional.

2. We call the sine, tangent, or secant of an angle; the sine, tangent, or secant of the arch which measures that angle.

3. In every right-lin'd triangle, as ABC, the half of the sides are the sines of the angle to which they are opposite: For having described a circle thro' the three points A B C, it will appear that the half of the side BC, for example, is the sine of half the arch BC, (by *Prop. 5. Trig.*) which measures the opposite angle A, (by 20. 3. *Euc.*)

COROL.

C O R O L L A R Y.

From whence it follows, that in every triangle, as $A B C$, the sine of the angle A is to its opposite side $B C$, as the sine of any other angle, as B , to its opposite side $A C$. *Fig. 2.*

4. In every triangle, as $A B C$, rectangular at the point B , if one leg, as $A B$, be taken for the radius of a circle, or sine total, the other leg $B C$, will be the tangent of the angle A , and $A C$ its secant. *Fig. 2.*

5. To resolve a Triangle, so much ought to be known of the angles and sides, as may determine the triangle in such manner, that it cannot be changed, without changing one of the angles or one of the sides.

P R O P. XII.

The Resolution of rectangled Triangles.

1. In the triangle $A B C$, of which the angle B is right, if the line $A C$ be known, and the acute angle C , then we may say, by *Suppos. 3.* *Fig. 3.*

As the sine total, or sine of the angle B : to the line $A C$:: the sine of the angle C : to the line $A B$.

You must therefore multiply the line $A C$ by the sine C , and divide the product by the sine total, the quotient will be the line $A B$.

Or by Logarithms. Add the log. of the line $A C$ to the log. of the sine C , and from the sum subtract

subtract the log. of the sine total; the remainder will be the log. of the line AB. See the work both ways, making $C=40^{\circ} 6' 26''$.

Note, Sine C	--- 64422	Log. Sine of C	- - - 9.8090342
Line AC	--- 50	Log. of AC	--- 1.6989700
Product	- - 3221100	Sum	- - - 11.5080042
Sine total	- - 100000	Log. of Sine total	- - - 10.0000000
Quotient AB	32.2	Log. of 32.211 for AB	- 1.5080042

In the same manner for the side BC, we say, by *Supposit.* 3.

$$\text{Sine total} : AC :: \text{sine A} : BC.$$

2. In the rectangle triangle ABC, of which the angle B is right, and the angle C known, with the side BA opposite to it; to find CA, we say, by *Sup.* 3.

$$\text{Sine of the angle C} : \text{side AB} :: \text{sine of the angle B} : CA.$$

And to find the side BC, we say,

$$\text{Sine C} : AB :: \text{sine complement C} : BC.$$

3. In the triangle ABC, of which the angle B is right, and of which the side CB, with the base AC, are known, to find the angle A, we say,

$$\text{The base AC} : \text{sine of the angle B} :: \text{side, BC} : \text{sine of the angle A}.$$

And to find the side AB, we say,

$$\text{The sine total B} : \text{the base AC} :: \text{sine complement A} : \text{side AB}.$$

4. In the triangle ABC, of which the angle B is right, if the two sides are known; the angle A may be found by saying, by *Supposit.* 4. Fig. 6.

The side AB : the sine total :: the side BC : tangent of the angle A.

P R O P. XIII.

The Resolution of oblique-angled Triangles.

1. In the triangle ABC, if the side AC be known, and the angle C and B, Fig. 7.
the side AB may be found, by saying, by *Suppos.* 3.

The sine of the angle B : side AC :: sine of the angle C : side AB.

And because the angle A, is the supplement of the sum of the two others, to find the side BC, we say,

The sine of the angle B : side AC :: sine of the angle A : side CB.

2. In the triangle ABC, if the sides AB, BC, be known, and the angle A, which is not between the known sides, the triangle cannot be resolved, unless it be known whether the angle C be acute, or obtuse; because without altering the data, we may change the triangle ABC, into the triangle ABD; but if it be known that the angle C is acute, or D obtuse, then may the triangle be resolved by *Sup.* 3. by saying.

Side BC : sine A :: the side BA : sine C, or D.

And to find the side CA, we say,

Sine A : BC :: sine B : AC.

3. In

3. In the isosceles triangle ABC, if the legs are known, and the angle B between the equal legs, the two other angles which are equal to each other, may be also known, and the whole triangle resolved.

4. But in the scalenous triangle ABC, if the legs BA, BC, are known, and the angle B, we may know the sum of the angles A and C; and to find the value of each, use the following analogy.

The sum of the legs AB, BC : their difference :: the tangent of half the sum of the angles A and C : to the tangent of an angle, which being added to the said half, will give the greater angle.

From the point B describe a circle, whose radius let be the lesser leg BC, which will cut the greater leg in the point D; produce DB till it meets with the circumference of the circle, at the point E; so shall AE be the sum of the sides AB, BC, and DA their difference; the angle DCE will be right, as will also the angle CDF, if DF be drawn parallel to CE.

Demonst. The angle B being common to the two triangles BDC, BAC, the sum of the angles on the base DC, will be equal to the sum of the angles on the base AC; and as the angles on the base DC are equal, (by 5. 1. *Euc.*) each of them will be the half of the sum of the angles A and C; From whence it may be concluded, 1. That the line CE must be the tangent of half the sum of the angles A and C. 2. That the angle ACD (twice the difference of the angles at the base) of which DF will be the tangent, being added to the half of the sum of the angles A and C, will be equal to the angle C, and consequently, since (by 4. 6. *Euc.*)

AE

AE is to DA, as CE to DF, it is certain, that the sum of the sides BA, BC, is to their difference, as the tangent of half the sum of the angles A and C, is to the tangent of an angle, which added to the said half, will make the greater angle C.

5. If all the sides of the triangle ABC are known; from the greatest angle B, let fall the perpendicular BD, upon the greatest side AC, and it will form two rectangle triangles, the angles whereof may be resolved by the foregoing propositions; but to find D the point where the perpendicular falleth in the line AC, we use the following analogy.

The greatest side AC : the sum of the other two sides BA, BC :: the difference of the sides BA, BC, viz. GA : AF, which subtracted from AC, the point D is in the middle of the remainder.

It is easy to see how the figure is made.

Demonst. The line ABE is the sum of the sides AB, BC, and the rectangle EAG is equal to the rectangle CAF, (by Cor. 16. 6. Eucl.) therefore CA : the sum of the sides AB, BC :: AG : AF, (by 14. 6. Eucl.)

R E M A R K.

If in the figure ABCD, the angles BAC, BAD, BDA, BDC, be known, which are sufficient to determine the other angles, all the other angles may be known, without knowing the side AB; to which you may give what value you please, without changing the angles sought. Thus knowing any one side, the whole figure may be resolved.

P A R T



P A R T IV.

The Resolution of SPHERICAL TRIANGLES.

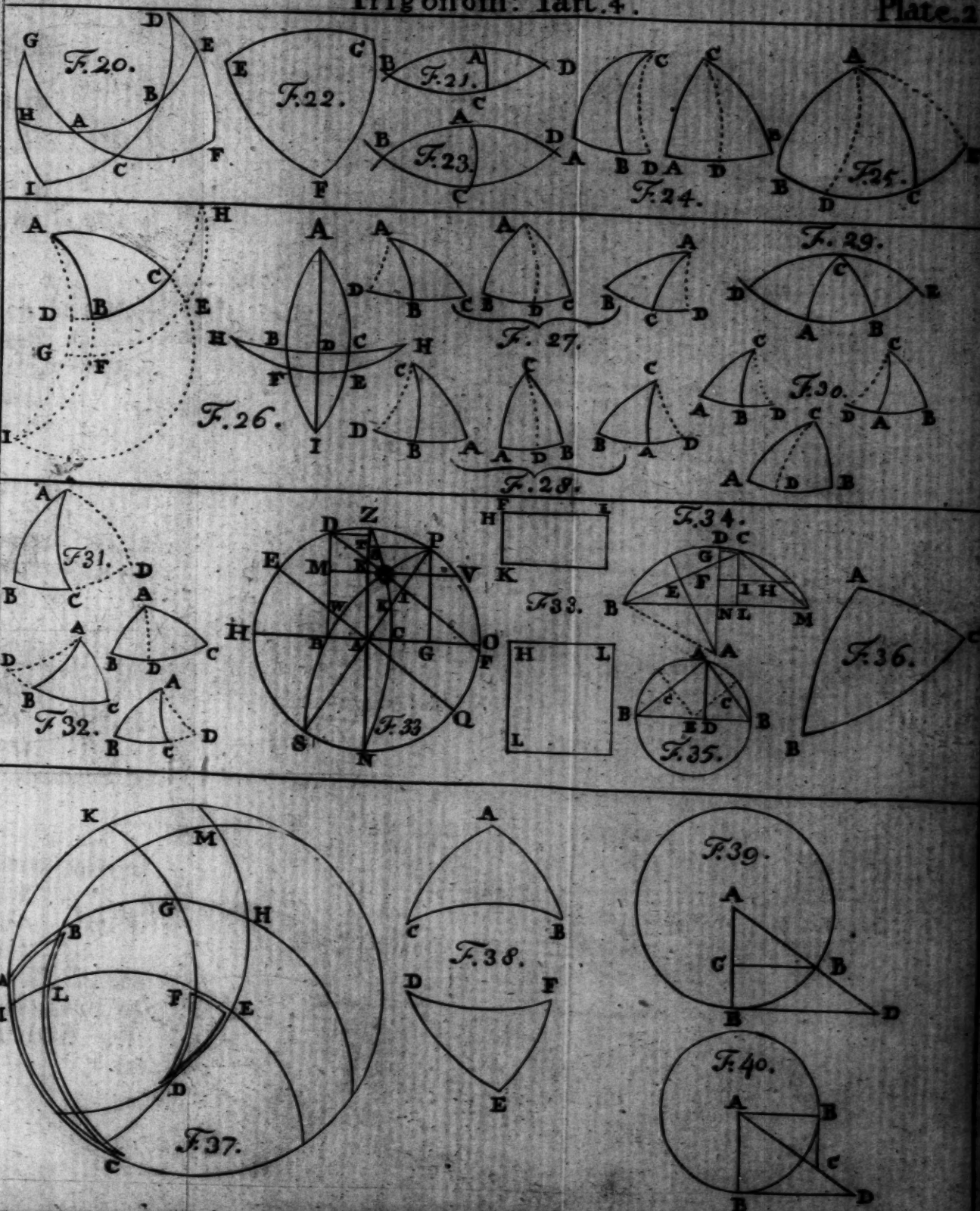
L E M M A I.

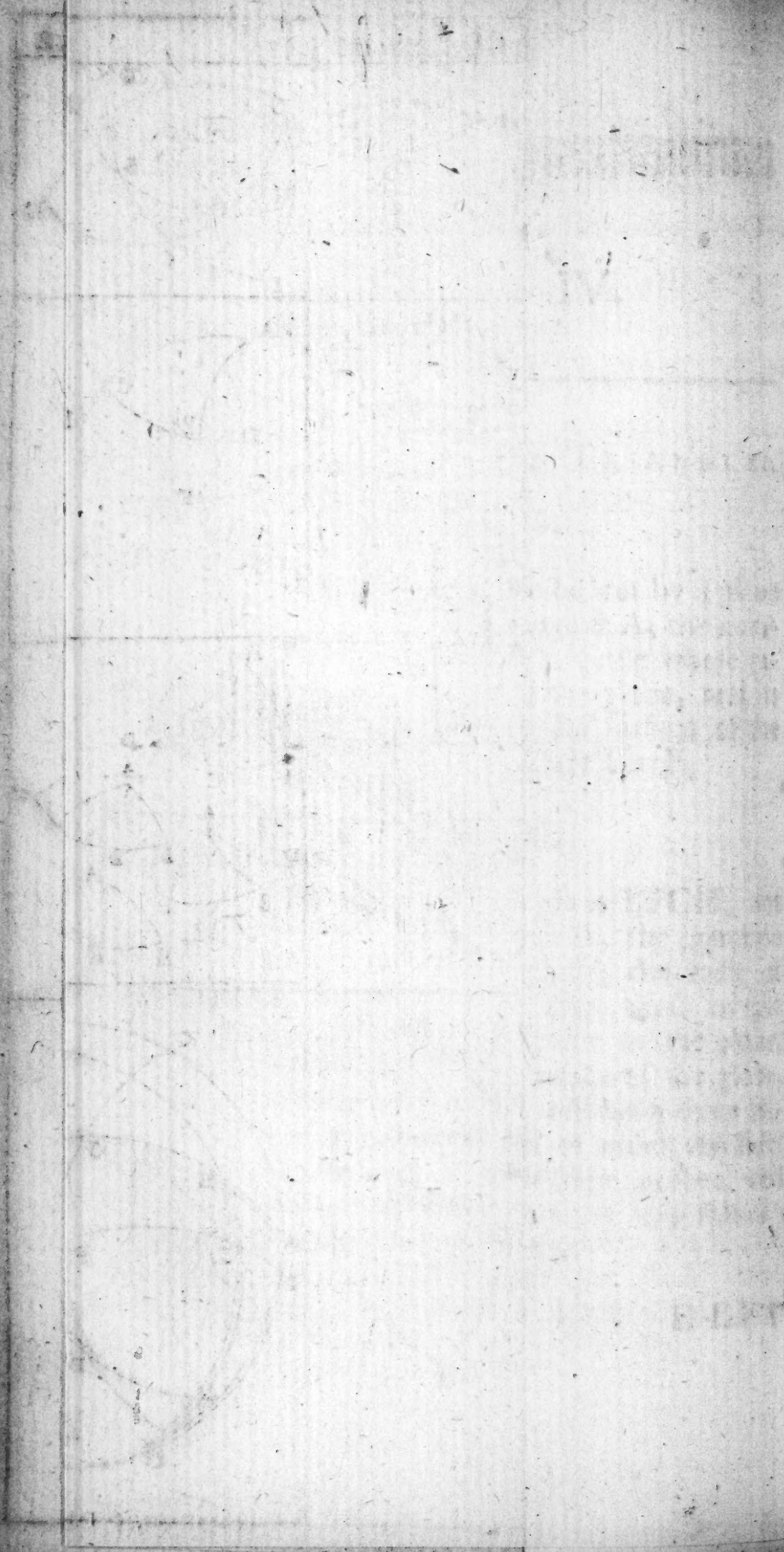
Fig. 1. IF a globe or sphere be cut by a plane passing thro' its center A, the common section will be a circle whose radius AB, will be the radius of the globe, and its circumference will encompass the surface of the globe; such circle is call'd a *Great Circle*.

L E M M A II.

If a sphere be cut by the plane EFGH, not passing thro' the center A, the common section will be a circle, the radii of which will be less than those of the sphere. Draw AB perpendicular to the plane, and BM, BN, BL, to the surface of the globe, and AM, AN, AL, and you will have three triangles equal in every respect, of which the sides BM, BN, BL, will be equal to one another, and less than AL, AM, AN, radii of the sphere; such circle is call'd a *Lesser Circle*.

D E F I





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sp
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wi
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eq

DEFINITIONS.

1. A spherical angle is form'd by two arches of a great circle, as ABC. *Fig. 3.*

2. A spherical triangle is a part of the surface of a sphere, bounded with *v.* *Fig. 3.* three arches of great circles, as ABC.

3. The pole of a circle is a point on the surface of a sphere, equidistant from every part of the circumference of the said circle. *Fig. 4.* Thus the points A and B are the poles of the circles CD and EF.

SUPPOSITIONS.

1. All spherical triangles are supposed to be made upon the surface of a sphere, of which the radius contains 10000 . . . equal parts.

2. If in a sphere two great circles ABC, ADC, cut each other in the points A, C, their common section will pass thro' the center of the sphere, which is common to both, and will consequently be the diameter of both, and will divide both equally. *Fig. 5.*

3. The line AB, which joins the two poles of the circle CD, is the diameter of the sphere. For imagining a great circle *Fig. 6.* to pass thro' the points A and B, which will cut the circle CID in the points C, D, the arches AC, and AD, BC, and BD, will be equal; since therefore the whole arch ACB, will be
be

be equal to the whole arch ADB, the line AB will be the diameter of the great circle CBDA, and consequently of the sphere.

Note, If CID is a great circle, CB and CA will be quadrants ; as will all other arches of a great circle drawn thro' the poles A, B, upon the circle CID.

4. Every spherical angle, as ABC, is measured by the arch AC of a circle, of which B *Fig. 7.* is the pole. This definition supposes, that the arches of all circles whose pole is B, will be similar to the arch AC, if they are between the arches BA, BC. From whence it follows, that the angles diametrically opposite B, D are equal, being measured by the same arch AC; since by the foregoing Supposition, the point D will be also the pole of the arch AC.

5. The arch BC falling upon the arch AE, makes two angles ACB, ECB, equal *Fig. 7.* to two right, since they are measured by the semicircle ABE, (by *Sup. 2.*) In the same manner the opposite angles BCE, ACD, will be equal, being each the supplement of the angle ACB.

6. If the circle ABC, passes thro' the pole A, of the circle BED, they will cut each *Fig. 8.* other at right angles. For describing a great circle, of which let B be the pole; it will pass thro' the points A, C; so that BA, BC, will be quadrants of a circle, (by *Sup. 3.*) but AE will also be a quadrant as well as EC. There-

Therefore the angles ABE, CBE, will be measured by quadrants.

7. If the circle BAD, makes right angles with the circle BED, they will also pass thro' each other's pole. *Fig. 8.*

8. If the point B, be not the pole of the circle DFE, but some other point, as A, the arch BAE will be greater than any other arch, as BF. *Fig. 9.*

Draw BC right upon the plane DEF, and it will fall upon DE, (by 38. 11.) at the point C, which will not be the center of the circle DFE; otherwise the lines BD, BE, would be equal. Draw BF, CF.

Demonst. The line CE passing thro' the center of the circle, will be greater than CF, (by 7. 3.) Then in the rectangled triangles ECB, FCB, which have the side CB common, the side CE being greater than the side CF, the base BE will be greater than the base BF; and because arches increase with their chords, till they become a semicircle, the arch BAE will be greater than BF.

9. The figure remaining as before, the arch BD will be less than any other, as BF, and BF will be greater than any other which can be made between D and F, and less than any other between F and E.

10. In every spherical triangle, ABC, each side, as AB, is less than a semicircle. *Fig. 11.*
For their arches AB, AC, being prolonged till they

they meet in the point D, the arch AD can only be a semicircle, (by *Sup.* 2.)

11. In every spherical triangle, as ABC, two sides AB, AC together are greater than the third BC. *Fig. 11.* From the point B, with the distance BA, describe the circle DAF, and compleat the circle BCGF.

Demonst. Since the point B is one pole of the circle ADEF, the point C, which is distant from it less than the space of a semicircle, (by the preceding) cannot be its other pole, (by *Sup.* 3.) but some other point, as G, if BG is a semicircle; then the arch CA is greater than CD, (by *Sup.* 4.) But BA and BD are equal, (by *Def.* 3.) therefore CA, and BA, are greater than CB.

12. The sum of the sides of a spherical triangle ABC, are less than two semicircles. *Fig. 12.* For continuing AC, AB, till they meet in the point D; their arches ACD, ABD, are only equal to two semicircles: but CD, BD, are greater than CB, (by the preceding); therefore AC, AB, CB, are together less than two semicircles.

13. If the spherical triangle ABC, has two legs BA, BC, equal to the two legs of another spherical triangle EF, FG respectively; and if the angle B, be equal to the angle F; the two triangles are equal in every respect.

Demonst. Let one triangle be laid on the other, then AB will cover FE, and BC will cover FG: therefore the base AC will cover the base EG: otherwise they would cut each other in the point E, G.

E, G, and each base would be a semicircle, (by *Sup. 2.*) which cannot be (by *Sup. 10.*)

14. It is demonstrable also in spherical triangles, as in rectilinear, that if the legs AB, AC, are equal to the legs ED, *Fig. 14.* DF; the angle included will be greater, or less, in the one triangle than in the other, according as their bases are greater or less. In the same manner it is demonstrable, that isosceles triangles have equal angles on their bases and that those which have equal angles on their bases, are isosceles triangles.

15. If in the triangle ABC, the angle A is greater than the angle C; *Fig. 15.* the side CB will be greater than the side AB.

Demonst. Make the angle DAC, equal to the angle DCA, and the arch AD will be equal to the arch DC, (by the preceding) but DB, DA, are greater than AB, (by *Sup. 13.*) therefore BD, DC, will be greater than AB.

16. In the triangle ABC, whose angle A is right, the angles C and B have the same affection as the sides opposite to them; *Fig. 16.* that is to say, they are right, acute, or obtuse, according as the opposite arches are equal to, or less, or greater than a quadrant.

Demonst. Since the arch AB is perpendicular upon the arch AC, it will pass thro' its pole, (by *Sup. 7.*) therefore if the arch AB is a quadrant, the point B will be the pole of the arch AC (by *Sup. 3.*) and consequently the angle C will be right (by *Sup. 6.*) but the arch AB cannot be augmented,

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mented,

mented, or diminished, without augmenting or diminishing the angle ACB: Therefore the angle C will be right, or greater, or less than a right, according as the arch AB is a quadrant, or greater, or less than a quadrant.

17. If the two semicircles ABC, ADC, cut each other at right angles; the point B, the pole of the semicircle ADC, will divide ABC into two quadrants, (by *Def. 2.*) In the same manner the point D, the pole of the semicircle ABC, will divide ADC into two quadrants. Farther, the arch DE will be a quadrant, (by *Sup. 3.*) and since E is not the pole of the arch ADC, the arch ED will be greater than the arch EF, and less than the arch EG, (by *Sup. 9.*)

C O R O L L A R I E S.

1. In every rectangled spherical triangle ADE the base DE will be a quadrant, if at least the leg AD is the fourth part of a circle: But if the two legs are greater than a quadrant, as in the triangle CEF, or if they are both less than a quadrant, as in the triangle AFE, the base EF will be less than a quadrant. Lastly, if one of the legs is less than a quadrant and the other greater, as in the triangle CEG, the base EG will be greater than a quadrant.

2. The three sides of a rectangled triangle cannot each of them be greater than a quadrant; nor can one side be greater than a quadrant, and the other two less.

3. The

3. The hypotenuse of a rectangle triangle, or the arch opposite to the right angle, cannot be less than a quadrant, without affecting the legs, and consequently the angles.

PROP. I.

If the spherical triangle ABC , having every one of its sides less than a quadrant, has the angle B right; the sine of the angle *Fig. 18.* B , will be to the sine of the opposite arch AC , as the sine of another angle A , to the sine of the opposite arch BC .

Continue BC , till BE becomes a quadrant, so shall the point F be the pole of the arch AB , which must also be continued till AE becomes a quadrant: then describing the arch FE , of which the pole must be A , and consequently the arch ACL will be a quadrant, and LE the measure of the angle A . If you draw LH right upon the circle ABE , it will fall perpendicularly upon the common section GE , and will be the sine of the arch LE , or of the angle A : LG will be the sine total, or sine of the angle B .

Farther, draw thro' the point C , a plane parallel to the circle ELF , and like it perpendicular upon the plane ABE . This plane will cut the plane, BCF , perpendicular upon ABE , and their common section CD will be right upon the same plane, and consequently parallel to the line HL , and the sine of the arch BC . In the same manner ID , and GH , CI , and CL , are parallel, (by 16. 11.) being the common sections of the two parallel planes with a third; and as LG is found perpendicular upon GA , IC will be also per-

pendicular upon GA , and the sine of the arch CA .

Demonstrat. The three sides of the triangle LHG , being parallel to the three sides of the triangle CID , the two triangles are equiangular, (by 10. 11.) therefore (by 4. 6.) LG , the sine of the angle B , will be to CI , the sine of the opposite arch AC , as LH , the sine of the angle A , to CD , the sine of the opposite arch BC .

LEMMA.

If the plane A , is perpendicular to the plane B , and CD perpendicular to the common section EH , it will be right upon the plane; since every line in the plane A drawn parallel to DC is perpendicular to EH .

PROP. II.

In the spherical triangle ABC , the tangent of the angle A , will be to the tangent of the opposite leg CB , as the sine total to the sine of the other leg AB .

Continue the arches AB , AC , BC , as above; then draw EI , the tangent of the arch DE , or of the angle A ; and BD , the tangent of the arch BC ; and by the foregoing *Lemma* they shall be right upon the plane ABE .

Farther, thro' the line BD , draw a plane parallel to the plane ELF , and like it perpendicular upon the plane ABE ; the common sections DM , BM , will be parallel to the common sections IG , EG ; and as EG is perpendicular upon FG , BM will be also perpendicular upon FG ; and will consequently be the sine of the arch BA .

Demonst.

Demonst. Since all the sides of the triangle BDM, are parallel to the sides of the triangle IEG; the two triangles are equiangular, (by 10. 11.) therefore EI, tangent of the angle A, will be to BD, tangent of the arch BC, as EG, the sine total to BM, the sine of the leg BA.

COROLL. I.

By these two Propositions, are all Cases of right-angled triangles resolved, in the manner following; provided all their arches are less than quadrants.

Let ABC be a triangle, whose angle C is right; continue CA to G, so that GC may be a quadrant, and G the pole of the arch BC, (by *Sup.* 3.) continue also BC, so that BI may be a quadrant, which will likewise make the arch GI a quadrant, (by *Sup.* 3.) and its pole B, and consequently BA being continued to the point H, will also be a quadrant. In the same manner continue CB, so that CD may be a quadrant, and AC, till AF becomes a quadrant; for then DF and AE will be also quadrants.

CASE I.

If the angle A, and the opposite leg BC, be known; the base AB may be found, by saying, by the first Proposition,

Sine of the angle A : sine of the leg BC :: sine total : sine of the base AB.

Or the leg AC may be found, by saying, by the second,

F 3

Tangent

Tangent of the angle A : tangent of the arch BC :: sine total : sine of the leg AC.

Sine total : tangent of BC :: Co-tangent of A : sine of AC.

The angle B may be also found, because the triangle BED will have its base BD, the complement of BC, which is known ; and its leg DE, the complement of EF, which may be known, because it is the measure of the known angle A. We say therefore, by the first,

Sine complement of BC : sine total :: sine complement of the angle A : sine of the angle B.

C A S E II.

If the angle A be known, and the adjacent leg AC, the other leg BC may be known, by the second proposition, thus :

Sine total : sine of the leg AC :: tangent of the angle A : tangent of the arch BC.

Also the base AB may be found, by the second, thus :

Sine total : sine ED, complement of the angle A :: tangent of the angle D, complement of the arch AC : tangent of the arch EB, complement of the base AB.

Or the angle B may be found, by the first, thus :

Sine total H : sine of the base AG, complement of AC :: sine of the angle A : sine of the arch GH, complement of the angle B.

C A S E III.

If the angle A, and the base AB be known, the leg BC may be found, by the first, thus :

Sine total : sine of the base AB :: sine of A : sine of the leg BC. Or

Or the other leg AC may be found, by the second, thus :

Sine DE, complement of the angle A : sine total : : tangent BE, complement of AB : tangent of angle D, complement of AC.

Or thus :

Sine total : sine DE, complement of the angle A : : tangent AB, complement of BE : tangent AC, complement of the angle D.

And the angle B may be known by the second, thus :

Sine total : sine AH, complement of AB : : tangent of the angle A : tangent HG, complement of the angle B.

C A S E IV.

If the legs AC and BC are known, the base AB may be found, by the first, thus :

Sine total H : sine GA, complement of AC : : sine G, complement of BC : sine HA, complement of AB.

The angle A may also be found by the second, thus :

Sine of the arch A C : sine total : : tangent of the arch BC : tangent of the angle A.

In the same manner may be found the angle B.

Sine of the arch BC : sine total : : tangent of AC : tangent of the angle B.

C A S E V.

If the leg AC, and the base AB, are known, the other leg BC, may be found by the first, thus :

Sine of the arch GA, complement of AC : sine total H : : sine AH, complement of AB : sine G, complement of BC.

The angle A may also be found by the second, thus:

Tangent of the angle D, complement of AC : tangent of BE, complement of AB :: sine total : sine DE, complement of angle A.

And the angle B may be found by the first, thus,
Sine of the arch AB : sine total C :: sine AC : sine angle B.

C A S E VI.

If all the angles are known, the base AB may be found by the second, thus :

Tangent of the angle A : tangent of the arch HG, complement of the angle B :: sine total : sine HA, complement of the base AB.

Or the leg AC may be found by the first, thus,
Sine of the angle A : sine of the arch HG, complement of angle B :: sine total : sine AG, complement of AC.

In the same manner may be found the leg BC.

C O R O L L. II.

Such right-angled spherical triangles as have two sides greater than quadrants, are resolved as follows. Let the triangle ABC be proposed, whose two sides AB, BC, are greater than quadrants ; produce the said two sides till they meet at the point D, which will give another triangle CAD, whose angle D will be equal to the angle B, and the side AC common to both ; the other angles, and the other sides, will be the supplements to the other angles and sides of the triangle ABC ; so that it will be easy to resolve the

the triangle $A B C$ by the triangle $A C D$.

If the rectangled triangle EFG , has the sides EF , EG , quadrants, the angles G and F will be right; and the arch FG the measure of the angle E , (by *Sup.* 17.) Fig. 22.

Note, Some resolutions will be equivocal, because the sine of an arch, and its supplement are the same; in such cases, the triangle cannot be truly resolved, unless we know the affection of the angle, or of the arch found, by means of the sine. Thus the triangle ABC cannot be resolved, tho' the angle A , and the arches AC and BC are known, unless we also know the affection of the arch AB , all the rest agreeing as well with the triangle DCA . Fig. 23.

P R O P. III.

In every oblique-angled triangle, as ABC , the sine of the angle A , is to the sine of the opposite arch BC , as the sine of the angle B , to the sine of the opposite arch AC ; Draw the perpendicular arch CD , which may fall either within, or without the triangle. Fig. 24.

Demonst. Since the triangle ADC is rectangled (by *Prop.* 1.) the sine total D : to the sine of AC :: sine A : sine of CD , and consequently, the rectangle under the sine total, and the sine CD , is equal to the rectangle under the sine A , and the sine of AC , (by 15. 6.) in the same manner, the triangle DBC , being rectangled, the sine total D : sine CB :: sine B : sine CD , from whence

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whence it will again follow, that the rectangle under the sine total, and the sine CD, will be equal to the rectangle under the sine CB, and the sine B, as it was equal to the rectangle under the sine AC, and the sine A; since then the rectangle under the sine AC, and the sine A, is equal to the rectangle under the sine CB, and the sine B; therefore (by 15. 6.) the sine A : sine CB :: sine B : sine AC.

LEMMA.

If in the spherical triangle ABC, the arch which falls from the angle A, perpendicularly upon the base BC, be found within the triangle, as AD, then the angles B and C will have the same affection as the arch AD, (by *Sup.* 16.) But if the perpendicular arch falls without the triangle, as AE, then the angle B, and the supplement of the angle C, viz. ACE, will have the same affection as the said arch AE, and so the angles B and C prove diversly affected.

COROLLARY.

By this proposition therefore oblique-angled triangles may be resolved, by thus letting fall perpendiculars, and thereby rendering them right-angled triangles. But their resolution will be rendered more easy by the proposition following.

PROP. IV.

Let the oblique-angled triangle ABC be proposed, of which the legs AB, AC are less than quadrants: draw the perpendicular arch AD, which will fall either within, or without the triangle, and produce it

till AG becomes a quadrant; then from the point A, as a pole, describe the quadrant GH, so that H may be the pole of the arch AG; the arches DB, DC, being prolonged, will pass thro' the point H, (by *Sup. 6.*) so that DBH, DCH, will be quadrants, as will also AE, and AF, and consequently CE, BF, will be complements of the arches AC, AB, as BH, CH, will be complements of the arches DB, and DC; and the arches FH, EH, will be complements of the arches GF or GE, or of the angles GAF, GAE. From the figure thus prepared, may be drawn the following conclusions; but observe, that the same figure must be made for the triangle BIC, of which the legs IB and IC are greater than quadrants.

CONCLUSIONS.

1. *The tangents complements of the legs, and the sines complements of the angles, which the perpendicular arch makes within the legs, are proportional.*

In the rectangle triangle BFH, the tangent of the angle H, is to the sine total, as the tangent of the arch BF, to the sine of the arch FH, (by *Prop. 2.*) In the same manner in the triangle CEH, as the tangent of the angle H, is to the sine total; so the tangent of the arch CE, is to the sine of the arch EH. Therefore, as the tangent of the arch CE, the complement of AC, is to the sine of the arch EH, the complement of the angle EAG; so the tangent of the arch BF, the complement of AB, will be to the sine of the arch FH, the complement of the angle FAG.

2. *The*

2. *The sines complements of the legs, and the sines complements of the arches between the legs, and the perpendicular arch, are proportional.*

In the triangle HBF, (by *Prop. 1.*) as the sine of the angle H, is to the sine total; so the sine of the arch BF, is to the sine of the arch BH. In the same manner, in the triangle CEH, as the sine of the angle H, is to the sine total; so the sine of the arch CE, is to the sine of the arch CH. Therefore, as the sine of the arch BF, the complement of the leg AB, is to the sine of the arch BH, the complement of BD; so the sine of the arch CE, the complement of the leg AC, will be to the sine of the arch CH, the complement of CD.

3. *The tangents of the angles of the base, are reciprocally proportional to the sines of the arches, between the perpendicular arch, and the legs.*

In the triangle BAD, (by *Prob. 2.*) the sine total : sine BD :: tangent B : tangent AD. Therefore the rectangle under the sine total, and the tangent AD, is equal to the rectangle under the sine BD, and the tangent B, (by 15. 6.) In the same manner, in the triangle ADC, the tangent AD : the tangent C :: sine DE : sine total, and consequently, the rectangle under the sine total, and the tangent AD, will be also equal to the rectangle under the sine DC, and the tangent C. Therefore the rectangle under the sine DC, and the tangent C, is equal to the rectangle under the sine BD, and the tangent B; and therefore (by 14. 6.) the tangent of the angle C : to the tangent of the angle B :: the sine of the arch BD : the sine of the arch DC.

I

4. *The*

4. *The fines of the angles made by the perpendicular arch, are proportional to the fines complement of the angles of the base.*

In the triangle BAD, (*Case 1 Cor. 1. of Prop. 1. & 2.*) the fine of the angle BAD, is to the fine complement of the angle B, as the fine total is to the fine complement of the arch AD. In the same manner, in the triangle CAD, the fine of the angle CAB, is to the fine complement of the angle C, as the fine total to the fine of the arch AD; therefore the fine complement of the angle B, is to the fine of the angle BAD, as the fine complement of the angle C, to the fine of the angle DAC.

COROLLARIES.

1. If in the triangle ABC, the angles A and B, with the side AB, be known, the triangle may be resolved, because the *Fig. 27.* Data are sufficient to determine it.

Imagine the arch AD, to fall perpendicularly from the angle upon the opposite side BC, in such manner, that the angle B, which is opposite to the arch AD, may be acute; this will produce three different cases, as they are express'd in the figure and which all have the same solution.

To find the angle C, look in the rectangled triangle ABD, wherein you know the acute angle B, and the hypotenuse AB; look, I say for the angle BAD, (by *Case 3. of Cor. 1. Prop. 1. & 2.*) afterwards, say (by *Conclusion 4.*)

The fine of the angle BAD : fine complement of the angle B :: the fine of the angle DAC : to the fine of the angle C.

To

To find the side AC, say (by *Conclusion 1.*)

Sine complement of the angle BAD : sine complement of the angle DAC :: tangent complement of the arch AB : tangent complement of the arch AC.

In the same manner we may know BC, by letting the perpendicular arch fall on AC.

COROLL. II.

If in the triangle ABC, the angles A and B are known, with the side CB, let fall *Fig. 28.* the perpendicular arch CD, as above, which will give you the rectangle triangle BCD, of which you know the angle B, and the base BC; therefore you will have the angle BCD, (by *Case 3. of Cor. 1. Prop. 1. & 2.*) Afterwards, to find the angle BCA, say (by *Conclusion 4.*)

The sine comp. of the angle B : sine comp. of the angle A :: sine of the angle BCD : sine of the angle DCA ; then the sum or difference of the two angles will be the angle BCA.

To find the side CA, say (by *Prop. 3.*)

The sine of the angle A : sine of the arch BC :: sine of the angle B : sine of the arch AC.

To find the arch BA, seek in the triangle BCD, (by *Case 3. of Cor. 1. Prop. 1. & 2.*) the leg BD; then (by *Conclusion 3.*) say,

The tangent of the angle A : tangent of the angle B :: sine of the arch BD : sine of the arch AD.

Observe,

1. If the arch CD be the supplement of the arch CA, and if DC, DB, be continued till they meet in the point E, the arch CD will be also the supplement of

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of the arch CE; and consequently the triangle ACE will be isosceles, and the angles A and E equal; as will also A, D; therefore the different triangles BCA, BCD, will have the arch CB common, and two angles equal; so that to resolve the triangle ABC, it is not sufficient to know the angles A, B, with the sine CB, but the affections of the angles, and of the unknown sides, must be also known.

2. If in the rectangled triangle ACD, the base AC is greater than a quadrant, the angles ACD, CAD, will be diversly affected, (by *Sup. 17.*) and on the contrary; from thence it follows, that to determine the affection of the arch AC, and of the angle CAD, it is necessary to know the affection either of the one, or the other. The same may be said of the arch BA; for if in the rectangle triangle BCA, the base CA is greater than a quadrant, the legs BA, BC, will be of different affections, and the contrary.

COROLL. III.

If the two sides AB, AC, and angle A of the triangle ABC are known, draw the perpendicular CD from one of the un- *Fig. 30.* known angles, upon one of the known sides; then to know the angle B, you must find the arch AC, (by *Case 3.*) and (by *Conclusion 3.*) say,

Sine BD : sine AD :: tangent A : tangent B.

The same must be done for the angle C; then to have the arch CB, you say (by *Conclusion 2.*)

Sine comp. AD : sine comp. DB :: sine comp. AC : sine comp. CB.

COROLL.

COROLL. IV.

If in the triangle ABC, the sides AB, AC, and the angle B, are known, the tri-

Fig. 31. angle cannot yet be resolved, without knowing the affection of the angle C;

for supposing the arches AC, AD, are equal, and that the arch DC be prolong'd till it meets with the arch BA in the point B; then the different triangles ABD, ABC, will have two sides equal and one angle equal; but then, the angle ACB will be the supplement of the angle ADB; so that in knowing the affection of the

Fig. 32. angle C, the triangle may be determined. To know then the angle C, say (by *Prop. 3.*)

Sine AC : sine of the angle B :: sine of the arch AB : sine of the angle C.

Farther, to find the angle A, let fall the perpendicular arch AD, as in the first case, and look in the rectangled triangle ADB, the angle BAD : (by *Case 3. of Cor. 1. Prop. 1. & 2*) then say (by *Conclusion 1.*)

Tangent complement of the arch BA : tangent comp. of the arch AC :: sine comp. of the angle BAD : sine comp. of the angle DAC.

Lastly, to know the side BC, you must seek in the rectangle triangle ADB, the leg BD, (by *Case 3. of Cor. 1. Prop. 1. & 2.*) then say (by *Conclusion 2.*)

The sine comp. of the leg AB : sine comp. of the leg AC :: sine comp. of the arch BD : sine comp. of the arch DC.

P R O P.

PROP. V.

In any spherical triangle $ZP\odot$, the rectangle contained under the fines of the legs $ZP, Z\odot$, is to the square of the radius, as the difference between the versed sine of the base or third side $Z\odot$, and the versed sine of the difference *Fig. 33.* of the legs to the versed sine of the angle contained by those legs.

For let $HZON$ be a projection of the sphere on the plane of a great circle, whereof ZP is a part, and let PAS and EAQ be two diameters, the former passing thro' the angular point P , and the other perpendicular thereto; also let ZAN and HAO be two diameters whereof the former passes through the angular point Z and is perpendicular to the latter. Let DF be drawn through the other angular point $\odot$ parallel to EQ and let the perpendiculars $PG, PS, DB, DT, V\odot M, IL$, and $\odot K$ be drawn; then AG (PS) will be the sine and PG the co-sine of ZP , DI the sine and AI the co-sine of $P\odot$ or $PDAB$ (DT) the sine and DB (AT) the co-sine of DZ , EW the versed sine and AW the co-sine of the angle P , VR the sine and AR the co-sine of $Z\odot$, and ZT the versed sine of DZ . Then by similar triangles AE (AR): AG :: $D\odot$: DM ; and by the property of the ellipsis AE : DI :: EW : $D\odot$; whence by compounding the two proportions AE^2 : $AG \times DI$:: EW : DM ; and consequently $AG \times DI$: AE^2 :: DM : EW .

COROL.

COROLLARY.

As AG the sine of one leg : AE the radius : :
 EW the difference between the versed sine of the
 base DO and the versed sine of the difference of
 the legs : to a fourth sine. Then DI the sine of
 the other leg : AE the sine total : : the fourth sine
 found : EW the versed sine of the angle DPO.

LEMMA I.

If AB is the radius of a circle, and BE the
 sine of half the arches BD, DC ; if,
Fig. 34. farther, we take CM, the difference of
 the arches BD, DC ; I say, that the
 sine total shall be to BE, the sine of half the arches
 BD, DC, as CH, the sine of half their difference,
 to CI, the half of CL, or GN, which is the dif-
 ference between the versed sine of the great arch
 BD, and the versed sine of the lesser one DC.

Demonstrat. Since the angle EAB, which is
 measured by half the arch BC, is equal (by 20. 3.)
 to the angle CMB or IHC, for CI being the
 half of CL, and CH the half of CM, IH will
 be parallel to ML (by 2. 6.) the two rectangles
 ABE, IHC, will be similar ; and consequently
 (by 4. 6.) $AB : BE :: CH : CI$. From whence
 we conclude, that the rectangle contain'd under
 the sine of the half of two arches, and the sine of
 half their difference, will be always equal to the
 rectangle contained under the sine total, and half
 the difference of the versed sines of the same arches.

L E M M A II:

The square of AC, which is the sine of half the arch AB, is equal to the rectangle under half the radius, and under the versed sine DB. Draw the perpendiculars AD, EC. *Fig. 35.*

Demonstrat. The rectangled triangles ADB, ECB, having the angle B common, will be similar; therefore (by 4. 6.) $EB : BC :: BA : BD$, or the half of $EB : BC :: BC$ half of $AB : BD$, (by 16. 6.) &c.

P R O P. VI.

In every triangle, as ABC, the rectangle under the sines of the arches BA and CA, is to the square of the sine total, as the rectangle under the sine of half an arch composed of the base BC, and of the difference of the arches BA and AC; and under the sine of half the difference between the base, and the difference of the arches AB and AC, to the square of the sine of half the angle A. *Fig. 36.*

Demonstrat. By the foregoing, the rectangle under the sines of the arches AB and AC, is to the square of the sine total, as the difference between the versed sine of the base BC, and the versed sine of the difference of the arches AB and AC, to the versed sine of the angle A. Put in the place of these two last lines, two rectangles, which shall have the sine total for their base, and these two lines for their height, or the half of these rectangles, and you will find that the rectangle under the sines of the arches AB and AC, will

will be to the square of the sine total, as the rectangle under the sine total, and under half the difference between the versed sine of the base BC, and the versed sine of the difference of the arches AB and AC, is to the rectangle under half the sine total, and the versed sine of the angle A. But by *Lemma 1.* this third rectangle is equal to the rectangle under the sine of half an arch composed of the base BC, and the difference of the arches AB and AC, and under the sine of half the difference between the base BC, and the difference of the arches AB and AC; and by *Lemma 2.* the fourth rectangle is equal to the square of the sine of half the angle A. Therefore the rectangle under the sine of the arches AB and AC, &c.

C O R O L L A R Y.

To find the angle A of the triangle ABC, of which all the sides are known.

1. *Add the logarithms which answer to the sines of the two arches AB, AC.*
2. *Double the logarithm of the sine total.*
3. *Add the logarithm which answers to the sine of half an arch composed of the base BC, and of the difference of the arches AB, AC, to the logarithm which answers to the sine of half the difference between the base, and the difference of the arches AB, AC.*

Then will you have three terms, of which the first being subtracted from the sum of the two others,

others, half the remainder will be the logarithm answering to the sine of half the angle A.

Note. If the sides AC, AB, were equal, we should draw a perpendicular upon the base, which would divide it equally.

P R O P. the Last.

Let the triangle ABC be proposed, wherein A is the greatest angle, and is also the pole of the circle KFD; let B be the pole of the circle HED, and C the pole of the circle IE. These three circles will form the triangle FED, of which the side FD, opposite to the angle A, will be its supplement; the side FE will be the measure of the angle C; and the arch DE the measure of the angle B. Compleat the circle AC, and all the other arches, till they terminate in the said circle.

Fig. 37.

Demonstrat. Since the points A, B, C, are the poles of the circles KD, HD, IE, the angles K, L, M, G, H, will be right, (by *Sup. 8.*) and for the same reason the point D, E, F, will be the poles of the arches AB, BC, AC; from whence it follows, that DG will be a quadrant, (by *Sup. 4.*) as FK; and GK, which measures the supplement of the angle A, will be equal to FD. In the same manner EL, and FI, will be quadrants, and consequently IL, which measures the angle C, will be equal to the arch FE. Lastly, DH, and EM, will be quadrants; and consequently, HM, which measures the angle B, will be equal to the arch DE.

Farther, the angles of the triangle FDE, will be measured by the opposite sides of the triangle ABC;

ABC ; only that in the place of the greatest angle E, its supplement must be taken.

Demonst. The arch HB, and the arch AG, are equal, being both quadrants, (by *Sup.* 2.) therefore GH, which measures the angle D, is equal to the side AB, &c.

C O R O L L A R Y.

If in the triangle ABC, the three angles are known, I suppose, in opposition to it, *Fig. 38.* another triangle DEF, wherein making DF the supplement of the angle A, I give to FE the value of the angle C, and to DE the value of the angle B. Then searching, by the foregoing, the angles D, E, F, the angle E gives me the supplement of the side BC, the angle D, the value of the side AB, and the angle F, the value of the side AC.

Observe, throughout all Trigonometry,

That as those analogies which have the sine total for their first term, are most commodious for working, it will be convenient to change such analogies as have it not, into others which have, which may be done by means of the following reflections.

1. The sine total AB, is a mean proportional between the sine complement AC, and the secant AD. *Fig. 39.* If therefore the following analogy were proposed,

*The sine complement of the latitude of the place :
sine total : : sine of the sun's declination : sine
of his amplitude.*

I should change it into this,

*Sine total : secant of the latitude : : sine of the
declination : sine of the amplitude.*

2. The sine total AB, is a mean proportional
between the tangent BD, and the tan-
gent complement BC, since the tri- *Fig. 40.*
angles ABC, ABD, are similar. If
then the following analogy were proposed,

*Tangent of the greatest declination of the sun :
sine total : : the tangent of the present declina-
tion of the sun : sine of its distance from its
nearest equinoctial point,*

I should change it to this,

*Sine total : tangent complement of the greatest
declination : : &c.*



PRACTICAL



PRACTICAL GEOMETRY.

Y PRACTICAL GEOMETRY, is here meant in general, the art of measuring Lines, Surfaces, and Solids.

PART I.

Of LINES.

RIGHT lines, or lengths, are measured by comparison with other right lines, whose lengths are known or determined, as a *Fathom*, a *Foot*, an *Inch*, &c.

PROP. I.

To measure Heights.

If from the point A, a plummet B be suspended by the line AB, and thro' the point D a line DC be drawn perpendicular to the line AB, the line CA will be what we call the height of the point A above the point D.

Fig. 1.

G

This

This idea of height, though not quite exact, as shall be shewn in another place, is sufficient at present for us to conclude the height of any thing to be a line sensibly parallel to the thread of a plummet, in the place of observation.

2. Let it be proposed to take the height of the tower AD. Place the foot of your

Fig. 2. instrument, (which may be either a quadrant, or a semicircle divided into degrees, &c.) in any point of the horizontal line CD, whose length you may measure with your rule, then disposing your instrument so that the plummet may play freely on the limb, look thro' the two sights till you see the point A, the top of the tower; for then will the point A, and the two sights be in the same right line AC, and the thread of the plummet FE being parallel to the perpendicular height of the tower AD, the angle EFC will be equal to the angle CAD, (by 15. 1.) which will be alternatively opposite to it, and consequently the angle complement of the angle EFC will be equal to the angle C, the complement of the angle A. Measuring therefore the line CD with your rule, you may resolve the triangle ADC. (by *Trig.* 12.) according to the following analogy;

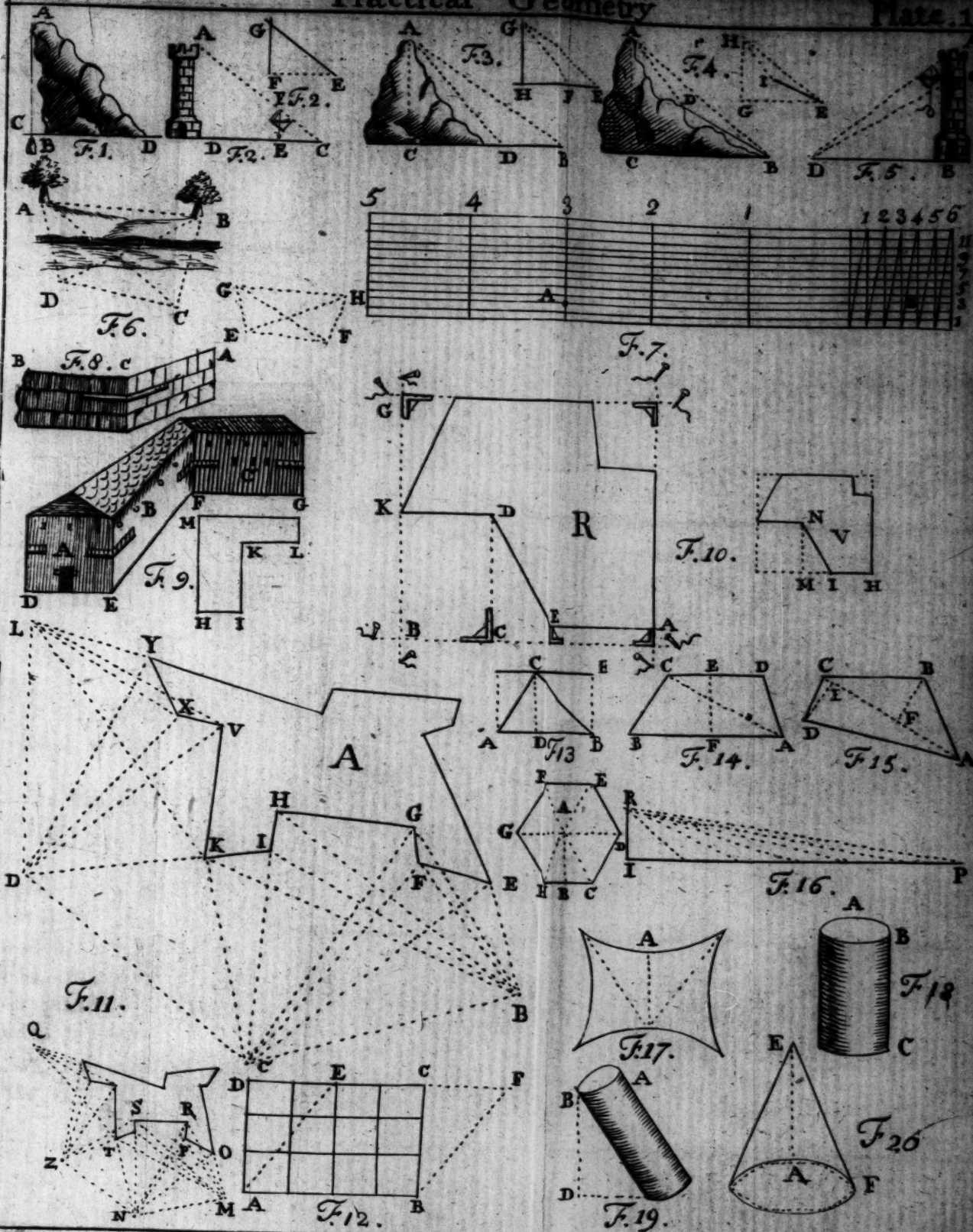
*As the sine of the angle A : to the line CD :
the sine of the angle C : to the line AD.*

In the same manner you may find the length of the line CA.

You may also resolve or measure the triangle ACD by protraction, thus : Lay down from any scale of equal parts, so many parts on the line
EF

Practical Geometry

Plate. 1



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EF, as you find feet or fathoms, &c. in the line CD, and drawing the perpendicular FG, lay down the line GE, so as to make the angle FEG equal to the angle C; for then the triangle EFG being similar to the triangle ACD, you may conclude, that there are as many feet or fathoms, &c. in the line AD, as you find parts of the line FE, in the line FG; so that the line FE will serve as a scale, to measure all the lines of the triangle ADC.

3. If you were to measure the height of the rock AC, above the surface of the earth B, and could not apply your rule *Fig. 3.* to the horizontal line BC. Take two points, as B, D, any where in the line BC, so that you may measure the distance BD with your rule, and can see the point A thro' the sights of your instrument; then at the point B take, as above, the angles BAC, ABC, and at the point D the angles DAC, CDA, which will furnish you with the following analogies:

As the sine of the angle BAD : to the line BD :: the sine of the angle DBA : to the line AD.

Then, *As the sine of the angle C : to the line AD :: the sine of the angle CDA : line AC.*

Or geometrically; laying down upon the line EF so many equal parts as there are feet, &c. between B and D, and making the angle GEF equal to the angle B, as the angle HFG must be equal to the angle D; then from the point G, where the lines EG, FG meet, draw the perpendicular GH, and the figure HFEG, will be similar to the figure CDBA, and the line FE will

will serve as a scale for the measurement, that is, all the lines of the figure HFEG may be measured by the scale from which the line FE was laid down.

4. If you cannot find two points in the same level, chuse two points B, D, whose distance you can measure at least with a cord, and from both which you may see the point A, thro' the sights of your instrument; then from the point B, take the angles DBC, ABC, BAC, and from the point D, the angles DAC, ADB, and you will have no trouble to resolve the figure ADBC, or to make a similar one EIGH, of which EI will be the scale.

5. In the same manner may be measured the lines AD, AB, from the top of a tower, provided we know the line AC, or DB, or DC.

PROP. II.

To measure inaccessible Distances.

Suppose you would know the distance betwixt two trees A, B, standing on the other side of a river: Chuse two stations, as C, D, the distance between which you may measure; then planting your instrument at D, and taking care it lies truly horizontal, look thro' the sights till you see the points C, A, B, so shall you find the angles ADB, BDC; do the same at the point C, to find the angles BCA, ACD, then will you be furnish'd with sufficient data

data to resolve the figure ABCD, or to make a similar one GHFE, measurable by the same scale from which you laid down the line FE.

P R O P. III.

To lay down the Ichnographical Plan of a place upon the spot.

An ichnographical plan is a figure similar to that which the thing itself makes upon the earth's surface. Thus the plan of a house is a just representation of the figure such house makes upon the level whereon it stands.

1. Provide a scale divided into what parts you please, as suppose fathoms, feet, and inches, as in Fig. 7, making your divisions for fathoms greater or less in proportion, as you would have your plan large or small.

Fig. 7.

The use of this scale is very easy : For if I would, for example, take upon it 3 fathoms 4 feet 2 inches, I place one foot of my compasses on the point A, common to the line of 3 fathoms, and that of 2 inches, and extend the other foot to the point B, where the line of 2 inches cuts that of 4 feet, and the distance AB, will evidently be 3 fathoms 4 feet 2 inches. (*See in the Appendix the description and use of Diagonal Scales.*)

2. Several instruments are used for taking the angle made by two walls, but the best sort is that called a *Metragon*, made of two rulers, join'd so at C, as that they may open and shut ; round the point C is described

Fig. 8.

G 3

scribed

scribed a circle divided into degrees, and on the point C, or center, is fixed a small index, which, as the rulers open or shut, runs thro' the divisions, and shews exactly the distance they stand at.

The use of this instrument is, by applying one ruler to one wall, and the other to the other, either on the inside, or the out, and the index will shew the angle they make with each other.

3. Suppose it were required to take the ichnographical plan of the building ABC.

Fig. 9. Measure the length DE of the wall A, and taking the same measure from your scale, set it upon the line HI; then taking the length of the wall B, and the angle it makes with the wall A, set off from your scale the length of the wall B, upon IK, laying down the line IK, so that the angle HIK, may be the same that the wall A makes with the wall B; do the same with all the other walls, and you will produce the figure HIKLM, which will be the plan of the building ABC. The reason of which is evident.

4. But because the multitude of walls and angles to be taken in a large building,

Fig. 10. may cause some error to slip in and so hinder the closing of the figure, observe the following method: Suppose a plan were to be taken of the building R. Extend the chord AB, divided into fathoms, feet, and inches, along one side, as AE, the length of which you may transport from your scale upon HI, which you may continue or produce as an obscure line, to what length you please. Farther, having a large square, slide one of its sides along the chord, till the

the other exactly answers to the point D, then take from your scale the distances AE, and EC, and having laid them down upon HIM, set likewise the distance CD upon the perpendicular MN, and the point N in the plan will represent the point D. Do the same for all the other points, as you may see in the figure, so shall the figure V be the exact plan of the building R.

In this manner may the plan of a City, with its several streets, be easily taken.

P R O P. IV.

To take Ichnographical Plans at a distance.

Suppose a plan were to be taken of the place A. Chuse two stations, as B, C, as far from each other as you can conveniently- *Fig. II.* ly, (measuring the distance) from whence you may see the most angles of the place, as E, F, G, H, I, K, then fixing your instrument horizontally at the point B, take (by *Prop. 2.*) the angles EBC, FBC, GBC, HBC, IBC, KBC, do the same at the point C, for the angles KCB, ICB, HCB, &c. then taking upon your paper two points M, N, to represent the points B, C, make the angles OMN, PMN, RMN, SMN, &c. equal to the angles EBC, FBC, GBC, &c. and the angles ONM, PNM, RNM, equal to the angles ECB, FCB, GCB, &c. and the lines MO, NO, meeting at the point O, as the lines MP, NP, at the point P, &c. will form the figure OPRSXT, similar to the figure EFGHIK.

To proceed with the plan, you must take two other stations, D, L, which are represented in the

plan by two other points, as Z, Q, in making the angles TNZ , ZTN , equal to the angles KCD , DKC , and the angles ZTQ , QZT , equal to the angles LKD , LDK ; it will be easy thus to take as many stations as you please, doing in all respects as at the stations B, C, to take in the whole circumference of the place. Each side may then be measured by the scale, from which you took the lengths of your stationary lines.

Note, 1. It will be still more easy to take plans of fields, and of roads, where you only fix your instrument at some principal points, which will serve as stations for taking those between them.

2. When we are obliged to take the plan of any place upon the sea, we use the Compass for taking the angles.





PART II.

To measure SURFACES.

SURFACES, or superficies, are measured by comparison with certain squares, denominated from the measure of their respective sides, as square fathoms, or square feet, &c.

PROP. V.

To measure plane Surfaces.

1. To find the area or content of the rectangular parallelogram ABCD, multiply its base AB by its height AD, and the *Fig. 12.* product will give you the content, that is, the squares it contains : and as any other parallelogram, ABFE, is equal to a rectangle ABCD, of the same base and height, (by 35. 1.) its content will be found by multiplying its base by its perpendicular height, as before, tho' it be not a rectangle.

2. The triangle ABC, being the half of a parallelogram of the same base *Fig. 13.* and height, (by 37. 1.) to have its content you must either multiply its base by half its height,

height, or its height by half its base, or multiplying the whole base by the whole height, take half the product.

3. In the trapezium ABCD, the opposite sides AB, CD, are parallel, therefore multiply the half of AB, and the

Fig. 14. half of CD, by the height EF. But if they are not parallel, multiply the diagonal AC by half the sum of the perpendiculars DE, BF.

R E M A R K.

Thus Surveyors measure land, and thus Artificers measure their works, as pavements, roofings, and floors, &c. These propositions may also be applied to the Military Art, in finding what number of men compose a battalion or squadron; for by multiplying the number of men in rank by those in file, you have the answer.

4. To find the area of a regular polygon, as A; draw from the center A, the perpendicular AB, to one of its sides HC, which being multiplied by half the circumference, or sum of the sides, will give the content: For if IR be taken for the height of a triangle equal to BA, and the base IP be made equal to the circumference CDEFGH, the triangle IRP will be equal to the polygon A.

5. A circle may be considered as a polygon of an infinite number of sides, to which its radius is perpendicular; therefore its area may be found by multiplying its radius by half its circumference. And thus we should find the quadrature of a circle,

circle, (which has never yet been obtain'd) if we could exactly determine the proportion between the circumference and the diameter. We can only find, by calculation, that the circumference is to the diameter nearly as 22 to 7, or nearer, as 3,14159 to 1.

To measure a surface whose boundaries are some of them strait lines, and others curve, divide it into several triangles, which, *Fig. 17.* by reason of their smallness, may pass for rectilinear, without any considerable error.

P R O P. VI.

To measure curve Surfaces.

1. To find the surface of a rectangular cylinder, multiply the circumference of its base, by its height; for if you imagine the *Fig. 18.* curve surface of a cylinder cut open and extended to a plane, it will become a rectangular parallelogram, whose height is that of the cylinder, and its base a right line equal to the circumference of the cylinder.

If the cylinder A is not rectangular, you must multiply the circumference of the base by the line BD; because its surface *Fig. 19.* will be an infinite number of parallelograms, having all the same height BD, and whose bases all together will make up the circumference of the base of the cylinder.

2. If the circumference of the base of a right cone A, be multiplied by *Fig. 20.* half its side EF, it will produce the area of its surface; which is the sum of an infinite number

number of triangles, whose height is EF , and all whose bases together form the circumference of the base of the cone.

LEMMA I.

If a right cone be cut by a plane parallel to its base, the common section D will be a *Fig. 21.* circle, whose radius will have the same proportion to MA , the radius of the base, as CD to CA , or as DO to MA , (by 4. 6.) and as this section will be perpendicular to the axis CDA , (by 1. 11.) the part DC of the divided cone, will itself be a right cone.

COROLLARY.

If HG be supposed equal to the circumference of the circle A , and LF equal to the side CM ; if also EF be equal to the side CO , then will IEK , parallel to HG , be equal to the circumference of the circle D . For the circumference of the circle D , is to that of the circle A , as DO to MA , or CD to CA , or as IK to GH , which was supposed equal to the circumference of the circle A . Thus the triangle FIK , will be equal to the surface of the cone DC , and the trapezium $IKGH$, will be equal to the surface of the section DA .

In the same manner, if we suppose the base V , equally distant from the bases D , and A , its circumference will be found equal to the line RP of the trapezium, equally distant from the line HG , and the line KI . But the trapezium $GHIK$ is equal to a rectangular parallelogram
of

of the same height, having RP for its base; therefore the surface of the section of a cone, is equal to a rectangular parallelogram, whose base is equal to the circumference of its mean base V , and height equal to the side OM .

LEMMA II.

Suppose the semicircle AKG , touch'd by the line OB in the point C , and by the line AE in the point A ; further, suppose CB , CO , equal. Draw BE , ON , and CM , parallel to AG ; and LK , XF , parallel to AE ; if then the whole figure be supposed to be turned about the perpendicular LK , whose point L is the center of the semicircle, the line OE will describe the surface of a right cylinder, whose radius will be ON , or LA , and its height XF ; and the line OB will describe the surface of a section of a cone, the radius of whose mean base shall be CM , and its side OB ; I say, that these two surfaces will be equal. Draw LC .

Demonstrat. The rectangular triangles EOB , CML , are equiangular, for the angle BCM , which is equal to the angle B , (by 15. 1.) the complement of the angle O , is also the complement of the angle LCM . Then (by 4. 6.) EO , or $XF : OB :: CM : CL$, or AL ; and since the radii are in the same ratio as the circumferences, (by 1. 12.) XF , or $OE : OB ::$ the circumference of the mean base $CM : the circumference of the base AL . Therefore (by 15. 6.) the rectangle under OE , and the circumference of the base AL , which is the surface of the cylinder,$

der, is equal to the rectangle under OB, and the circumference of the mean base CM, which is the surface of the section of the cone.

If the point B falls in the line LK, the same thing will follow, tho' the cone be not cut. For the triangles XBN, CLM, will be similar, and consequently BN, or OE : XB :: CM : CL.

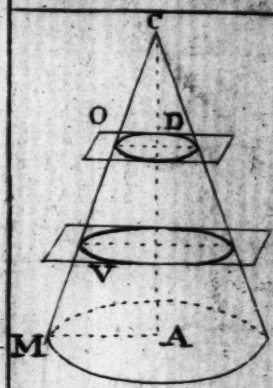
COROLL. I.

If about the semicircle A be circumscribed the polygon DBE, and the perpendiculars AB, DC, EF, be drawn upon the diameter DE; if, further, the whole figure be supposed to be turned about the line AB, the sides of the polygon will describe surfaces of cones, which all together will be equal to the surface of the cylinder described by the line DC. But the semicircle is a polygon of an infinite number of sides; therefore the surface of the semiglobe, which it describes, will be equal to the surface of the cylinder described by the line DC.

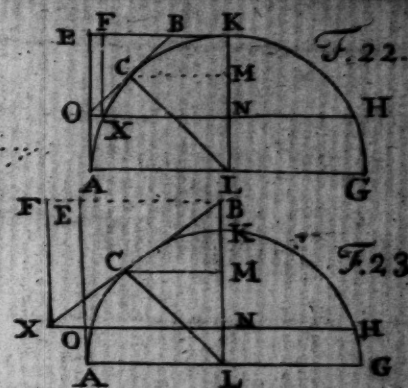
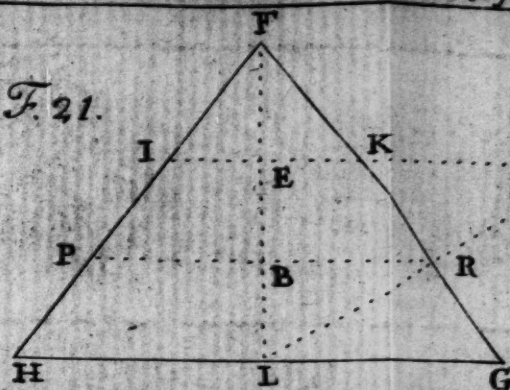
COROLL. II.

If the surface of the cylinder, and that of the semiglobe, be each of them divided into several parts by parallel planes, each part of the one, will be equal to the correspondent part of the other.

PROP.

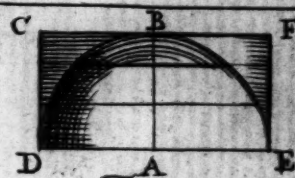


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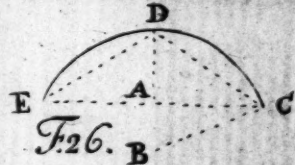


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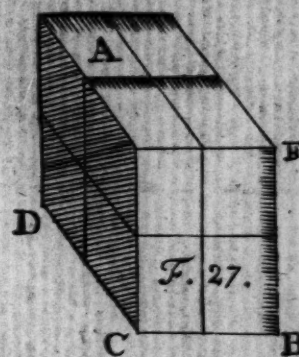
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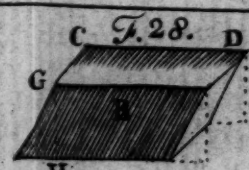
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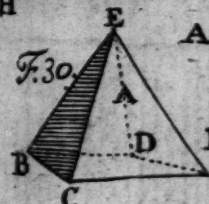
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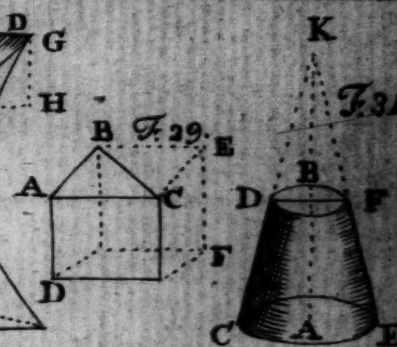
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F. 28.



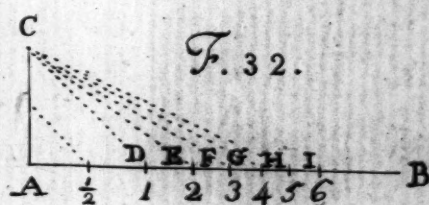
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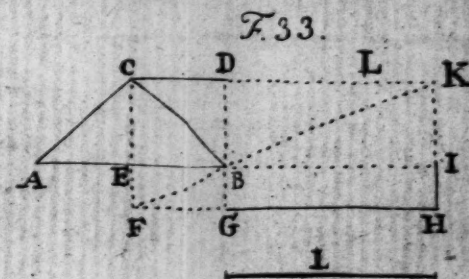
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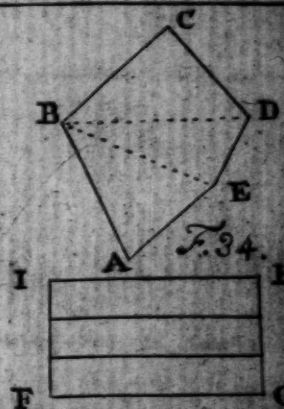
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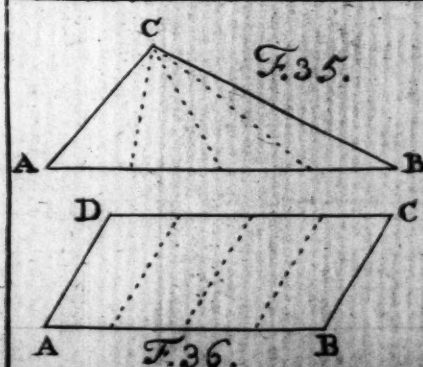
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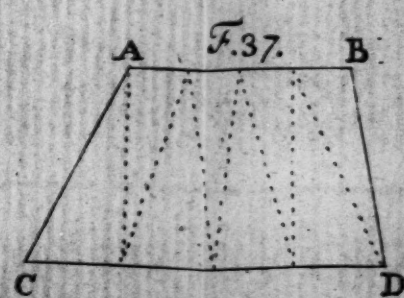
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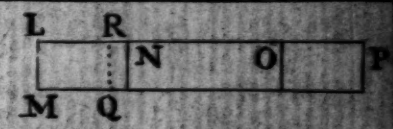
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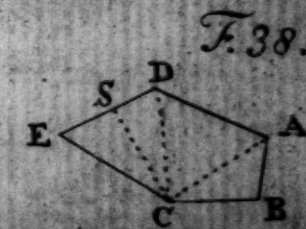
F. 35.



F. 36.



F. 37.



F. 38.

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P R O P. VII.

To find the surface of a semiglobe, multiply its radius by the circumference of one of its great circles, or by $\frac{22}{7}$ of its diameter. *Fig. 26.* In the same manner, to measure the concavity of a vaulted or spherical roof, pass a circle thro' its summit D, and thro' two points E, C, diametrically opposite; then multiply the height AD by $\frac{22}{7}$ of the diameter of such circle. All which follows from the preceding *Coroll.*

We take no notice here of surfaces described by the circumvolution of parabolas, hyperbolas, ellipses, &c. nor of bodies bounded by those figures, as not being necessary to our present design.





PART III.

To measure SOLIDS.

SOLIDS are measured by comparison with certain known cubes, denominated from their respective sides; as cubical fathoms, cubical feet, &c.

PROP. VIII.

1. To find the solidity or content of the rectangular parallelepipedon A; multiply the
Fig. 27. area of its base BCDE, by its height BF, because if its base contains 4 feet square, and its height two feet, it will contain two layers of 4 feet square each.

2. If the parallelepipedon B is not rectangular, you must still multiply its base by its
Fig. 28. height GH, because it is equal to a rectangular one of the same base and height.

3. To find the content of a prism, as A, multiply the area of its base by its height,
Fig. 29. it being the half of the parallelepipedon ABEC, which is to be understood of all kinds of prisms, as well as triangular ones,

ones, since they all may be divided into triangular prisms ; and further, since cylinders are prisms of an infinite number of sides, we also multiply the area of their bases by their height, to have their content.

4. The area of the base of a pyramid A, being multiplied by the $\frac{1}{3}$ of its perpendicular height AE, will give its content, *Fig. 30.* because it is but the one third of a prism of the same base and height (by 7. 12.) The same is to be understood of cones, which are pyramids of an infinite number of sides.

5. A globe may be considered as an infinite number of pyramids, whose vertexes meet in the center, and whose bases form the circumference : the radius of the globe will be the common height of the pyramids ; consequently therefore, multiplying the surface of the globe by $\frac{1}{3}$ of its radius, will produce its content, that is, the content of all the pyramids composing it.

6. In the same manner the sector of a globe is equal to a cone, having the radius of a sector for its height, and its base equal to the convex surface of the sector. Thus may a sector, or any other part of a globe be easily measured.

7. To measure the frustum of a cone, as CDFE, prolong its sides CD, EF, till they meet in the point K ; then measure the *Fig. 31.* cones CKE, DKF, and subtract the latter from the former, the remainder will be the content of the frustum CDFE.

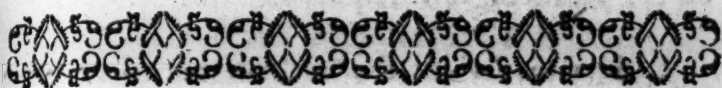
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To avoid a multiplicity of operations, the common practice is, to multiply the base A, and the base B, by the half of their distance A, B, tho' this way is not altogether exact.

In order also to find more readily the content of the bases by their diameters, we make
Fig. 32. use of a rule AB, divided in the following manner. Take AD equal to 1 foot 3 inches, and a little more than three lines, (which is the measure of the diameter of a circle equal to a foot square) and set AC equal to AD, perpendicular upon AB: it is then plain (by 47. 1.) that CD will be the diameter of a base of 2 feet; make then AE equal to CD, and AF equal to CE, and AG equal to CF, &c. and so will you obtain the several divisions of your rule; and in the same manner also may you lay down the halves, and quarters. Thus we measure Vessels, Masts, Yards, Piles, &c.

8. Other irregular bodies are measured by dividing them into prisms, and pyramids, and oftentimes the shortest ways are preferred to the exactest. Thus some gauge a ship as if it were a prism, by multiplying the length of of its keel into a triangle, whose height is the depth of the ship, and its base the mean width, or length of the middle ship beam.





PART IV.

Some PROBLEMS.

THIS part in the original contains five problems, the first and second of which being the construction of the *French* sector, and the proportional compasses, and the fourth the manner of inscribing regular polygons in a circle; as I have more largely treated of the first in the appendix to this work, and included the last in the extract I have added of the fourth Book of *Euclid*, omitted in the *French*; I shall therefore here only add the two following Problems.

PROBLEM I.

To make plane figures equal to others given.

1. To make a parallelogram equal to a triangle ABC, let its base EB be equal to half the base of the triangle, and make *Fig. 33.* it between the parallels EB, CD, (by 41. 1.) But if the parallelogram must have a determin'd line L for its base, continue CD, till DK be equal to L, then drawing the line KB, till it meet CE in the point F, and finishing the parallelogram

lelogram CKFH, its complement BGHI will be the parallelogram required.

2. If you divide the right-lined figure given ABCDE, into three triangles, and
Fig. 34. change those triangles into three rectangles having the same base FG, (by the preceding) placing them one upon another you will make a rectangle equal to the rectilinear figure given.

3. If you seek a mean proportional between the base FG of the rectangle FGIH, and its height GH, (by 13. 6.) the square upon such mean proportional will be equal to the rectangle FGHI, and consequently to the rectilinear figure ABCDE.

PROBLEM II.

Of the division of right-lined plane figures.

1. To divide a triangle into any number of parts, either equal or unequal, in any proportion given; divide its base AB
Fig. 35. into such number of parts, and in such proportion; then draw lines from the point C, to each division: so shall the triangle be divided in the proportion required, (by 1. 3.)

2. In the same manner to divide the
Fig. 36. parallelogram ABCD, divide its parallel sides AB, CD. The same is to be

Fig. 37. done with a trapezium, as ABCD, whose sides opposite AB, CD, are parallel.

3. If the rectilinear figure $ABCD$ be to be divided in any given ratio; divide it into triangles, and make three rectangles of the same height, equal to the three triangles, viz. LN , NO , OP , (by *Prob. 1.*) then divide the line LP according to the given ratio in the point R , and supposing that point falls in the base LN of the parallelogram, equal to the triangle CDE , divide DE in the point S , in the same ratio as LN in the point R , and the line CS will divide the rectilinear figure in the ratio given, as the perpendicular RQ divides the rectangle MP in the said ratio, (by the preceding) since the triangle CSE will be equal to the rectangle $LMQR$, both the one and the other having the same ratio.





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MECHANICS.

DEFINITIONS.

Y *Mechanicks*, is here meant the
B art of moving or raising heavy
bodies with facility.

2. We call *Gravity* that inclination whereby heavy bodies by their weight descend, or approach the center of the earth.

3. By *Motion* we understand the successive progression of a body from one place to another, and of the quantity of such motion we judge, 1. By the magnitude of the body moved. 2. By the length of the space it runs through in a limited time.

4. *Velocity* therefore is proportional to, and may be expounded by the space a body moves through in a given time.

5. *Power*, is that which gives the motion, whether it be an animated thing, as the hand of a man, or inanimate, as the weights of a clock.

PHYSICAL SUPPOSITIONS,

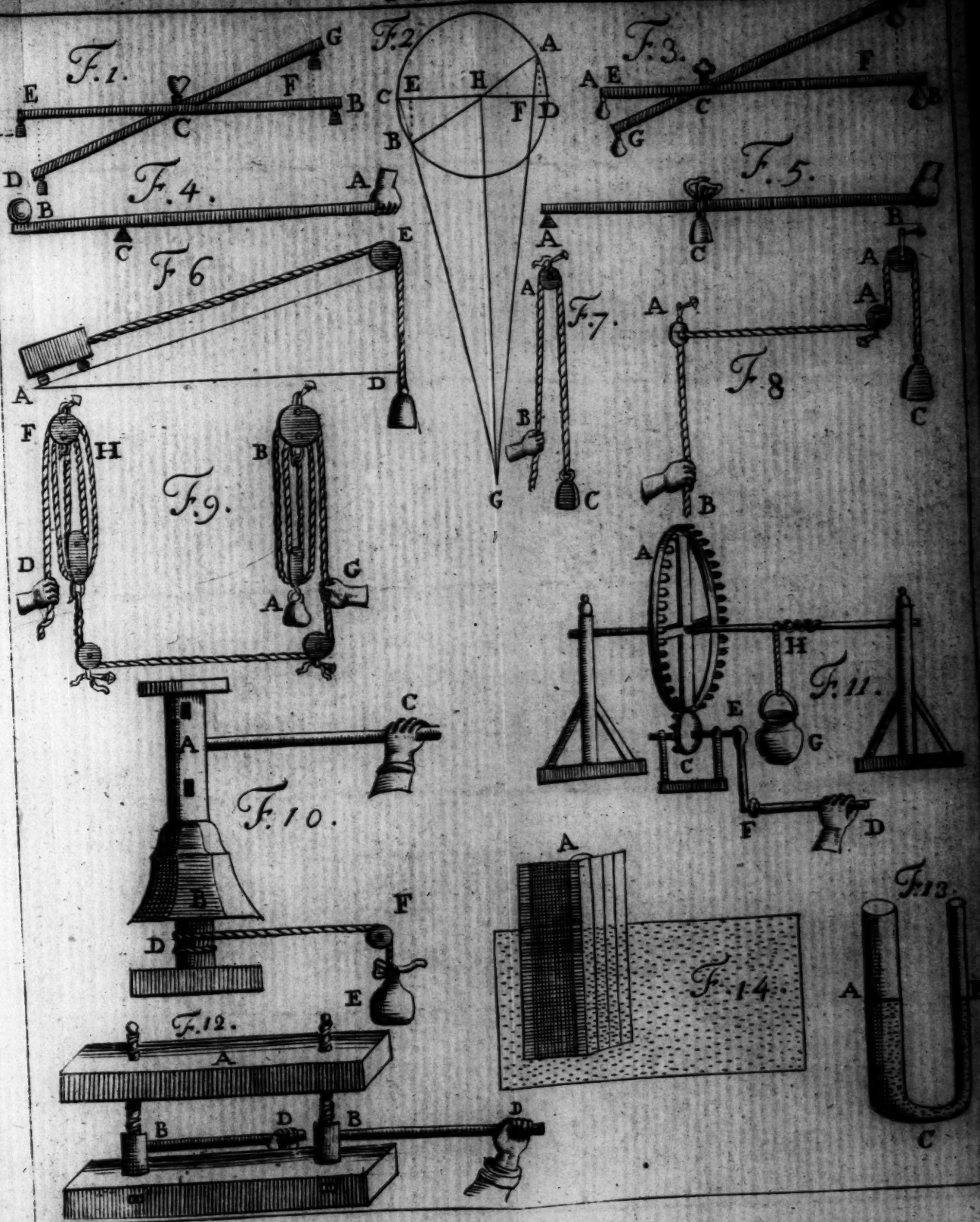
1. The resistance found in moving a body proceeds chiefly from two causes : 1. The body we would move has sometimes beforehand a contrary motion to that we would give it, as when we would raise a weight which has a natural tendency to descend. 2. At other times, the body we would move cannot stir without the partition or division of something containing it, as when we would move a body in the water.

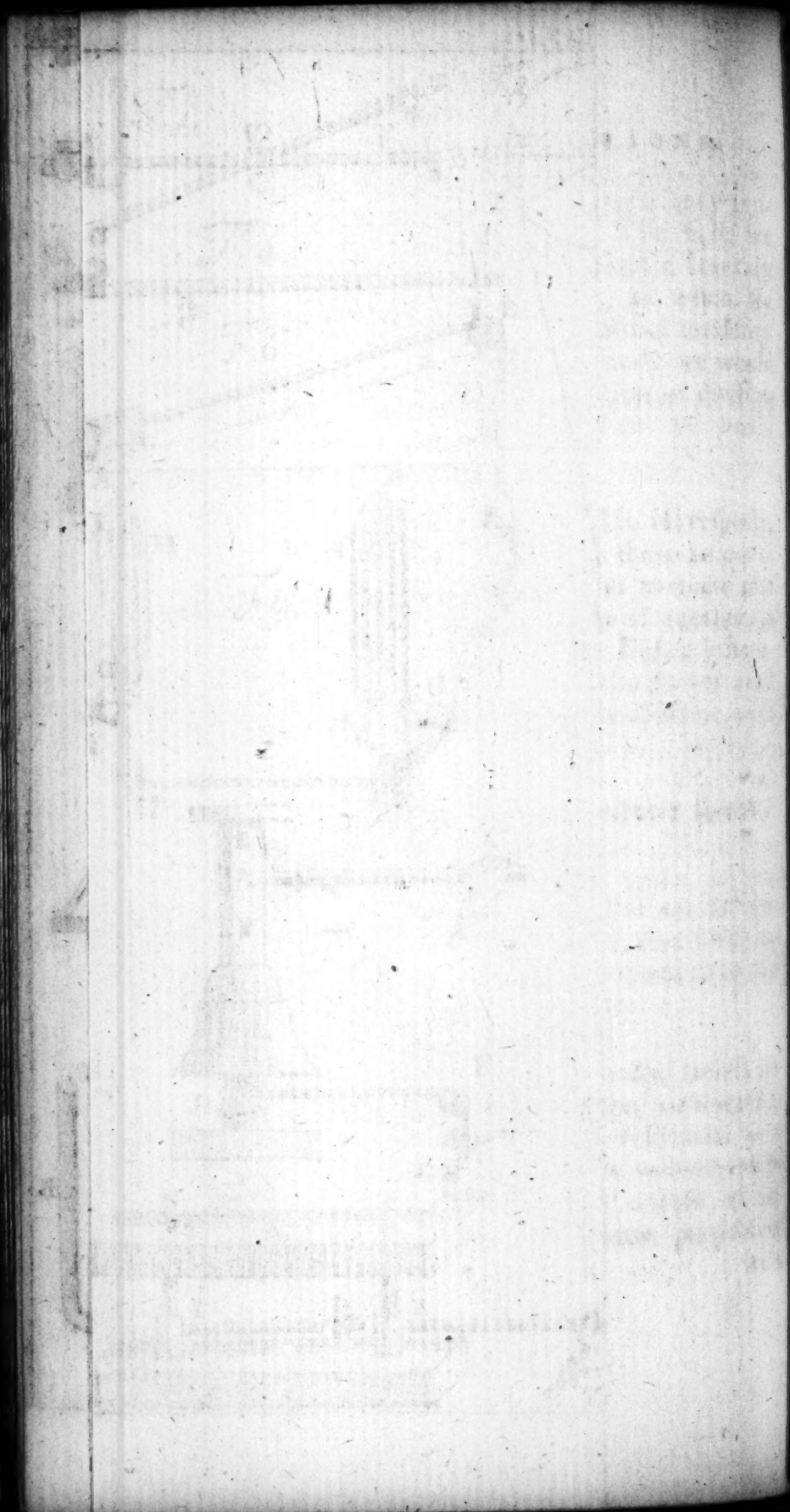
2. If two bodies at rest are equal in all respects, an equal power is necessary to give them an equal velocity of motion, (by *Def. 3.*) But to make one body move with twice the velocity of another, a double power must be applied, (by *Def. 3.*) therefore every thing else being equal, the power must be increased in the same ratio or proportion with the velocity.

3. A power can only communicate so much motion as is in itself.

4. Gravity being nothing else but an inclination or tendency towards the earth ; that force increases always in a reciprocal proportion to the square of the distance from the center.

5. A power cannot move a body, unless its force be greater than the resistance, or contrary motion of the body ; and as the resistance of a body increases in proportion to the velocity of its motion, (by *Def. 3.*) it is evident the force of the power ought to increase in the same proportion with





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with the velocity of the body, all other circumstances remaining equal. The following Axiom may therefore be looked upon as infallible.

A power is capable of moving a body, when the force of that power has a greater proportion to the contrary force of the body, than the velocity of the body to be moved has to the velocity of the power, and not otherwise.

Upon this principle then, (which exactly agrees with *Definition 3.*) the whole doctrine of Mechanics may be said to depend, without searching after any others, as many authors do to no purpose.

P R O P. I.

If the weights A and B are equal in all respects, and are hung on the two ends of a balance horizontally suspended at the point C, so that the parts CA, CB, are exactly equal, the weights will remain in equilibrium. *Fig. 1.*

Demonst. The weight A cannot descend to the point D, without raising the weight B to the point G, which is impossible, (by *Ax. preced.*) since its force has no greater proportion to the contrary force of B, than the space or velocity FG of B, has to the space or velocity, ED of A.

R E M A R K.

It is true, that as the descent or ascent of heavy bodies are measured by lines tending towards the center of the earth, as CB, *Fig. 2.* FA, the weight B will descend more than the weight A will rise. For $HG : GF :: H : AD :$

$AD : FA$, and $HG : GC :: EB$, or $AD : BC$; then since GC is greater than GF , BC will be greater than FA . But as this difference will cease upon placing the beam duly horizontal; so if the side on which the weight A hangs be inclined, then will the weight A have the advantage of the weight B , for the same reason.

2. If the weight A is double the weight B , and if reciprocally the distance CB is

Fig. 3. double the distance CA , the weights will be still in equilibrium. -

Demonst. The force of the weight A will be to the force of the weight B reciprocally, as the velocity or space run thro' of the weight B , viz. DF , to the velocity or space EG , run thro' by the weight A ; therefore (by *Ax.*) the weight A cannot raise the weight B , nor the weight B the weight A .

In this case it is also necessary, that the beam be perfectly horizontal, for the same reason as before.

3. It is easy to conclude, that if the velocity of the weight B increases in greater proportion than the velocity of the weight A , the weight B will consequently raise the weight A .

C O R O L L A R Y.

From what has been said it may be concluded, that the whole art of Mechanicks consists in diminishing the velocity of the weight to be moved, at the same time that we increase the velocity of the power causing such movement.

This may be done many ways, of which these that follow are the chief.

P R O P. II.

Of the Lever.

1. If the bar AB be so rested on the point C, that the part BC be shorter than the part AC, when the power A descends, *Fig. 4.* the weight B will be raised; but the velocity of the power A will be greater than the velocity of the weight B, in the same proportion as CA is greater than CB; therefore the resistance of the weight B will diminish in the same proportion with regard to the power A, (by *Cor. preced.*) that is to say, its resistance will become to what it was before, as CB to CA.

We call the bar AB in this case, a *Lever of the first kind.*

2. If the bar AB be rested upon the point A, and the weight be hung at C, its resistance with regard to the power B, *Fig. 5.* will diminish in the same proportion as CA is less than BA, because its velocity will be diminished in that proportion. For since the arch of the ascent of the power B will have AB for its radius, the arch of the ascent of the weight C, will have for its radius CA. In this second case, AB is called a *Lever of the second kind.*

It will be easy to refer to these two kinds of levers, several sorts of instruments, as pinchers, scissors, &c.

PROP. III.

Of inclined Planes.

If the weights A and D are fastened to the chord AED, and the weight D descends by the vertical line DE, whilst the weight A is raised up the inclined plane AE, in a direction parallel to that plane the velocity or space run thro' by the weight A, when come to E, shall be measured by the line ED, since that is the true height it is raised; and the velocity or space run thro' by the weight D, shall be measured by the line AE; and consequently the resistance of the weight A will diminish with regard to the power D, in proportion as DE to EA; that is to say, that upon the inclined plane AE, the resistance of the weight A, will be to its own natural resistance as ED to EA, supposing no impediment by friction.

Wedges, and nails are to be referred to inclined planes.

PROP. IV.

Of Pullies.

1. The pullies A being fix'd, do not at all diminish the resistance of the weight C, with respect to the power B, because they make no alteration in their velocity. On the contrary, the friction of the chord passing thro' the pullies, increases the resistance of the weight; therefore such fixed pullies are never used.

used but for the more easy application of greater powers.

2. To lessen the friction of the chords, wheels are placed in the pulleys; which the bigger they are, the better, because their circumferences have thereby the greater velocity with regard to the friction of the axis, besides that they make fewer turns.

3. If the weight A be fastened to the moveable block A, and a rope being fix'd to the immoveable block B, pass several times *Fig. 9.* thro' both, it is evident that the power C, being applied to the end of the rope, cannot descend without raising the block A; but to raise it the whole length of the line AB, the power must descend as many times the length of the same line, as there are returns of the rope from one block to the other. So that the velocity of the weight A, will be to the velocity of the power C, as one to the several turns of the rope; or as unity to twice the number of moveable pulleys.

4. It is easy to perceive how by this means the resistance of weights may be indefinitely diminished, *viz.* by multiplying the turns of the rope, or increasing the number of pulleys, as in *Fig. 9.* wherein the resistance is 4 times less with regard to the power C, which has 4 times its velocity, but becomes 16 times less with regard to the power D, which has 16 times a greater velocity.

5. For the same reasons, the weight A will have its whole force upon the rope which sustains the immoveable block B, because it has the same velocity with the block B; for if the block B be let loose, it will fall thro' just the same space as the weight itself would. But the weight A lays but $\frac{1}{4}$ of its stress upon the rope which fastens the end B to the block, because if the said end be unloosed, it will have 4 times more velocity than the weight; so also the rope fastening the end H to the block F, will bear but the $\frac{1}{16}$ of the stress of the weight, because that end will have 16 times the velocity of the weight.

PROP. VI.

Of Capstans, Wheels, &c.

1. When the power C, at the end of the bar AC, has turned the capstan AB, the rope DF will be also turned round the axle D, and will have so far advanced the weight E; but at every turn of the capstan the power describes the circumference of a circle, whose radius is AC, and the weight E is only so far advanced as is one turn of the rope about the axle D; and consequently the velocity of the power will be to the velocity of the weight, as the radius CA to the radius of the axle D, and the force of the power C will be augmented with regard to the weight E, in the same proportion.

2. It is easy to apply the same consideration to Wheels, to Cranes, &c. But *note*, the power of wheels may be indefinitely

nately increased by mixing pinions, or small wheels, among the great ones. For example, if the radius of the axle of the vertical wheel A, which has 72 teeth horizontally placed, is but a fourth part of the radius EF of the handle D, and if the wheel C has but 6 teeth, which take the teeth of the wheel A, when the power D shall have made 12 turns, the wheel A, and its axle, will have made but one, and the weight G will be raised but one turn of the cord about the axle HB; therefore the velocity of the power D will be to that of the weight G, as 12 turns of the handle to one turn of the axle H, or as 12 to $\frac{1}{4}$, or as 48 to 1; and consequently the power D needs only be the $\frac{1}{48}$ part of the weight G.

3. A variety of machines may be made by the multiplication of wheels; but great care must be taken that the friction be as little as possible.

P R O P. VII.

Of the Screw.

If the powers D turn the screws B, at each turn the weight A will be raised one notch, and the power will describe the circumference of a circle, whose radius will be BD; and consequently the resistance of the weight A, will be diminished with regard to the power D, in proportion as the height of the notch is to the circumference of the circle, whose radius, is BD.

Fig. 12.

PROP. VIII.

Of Liquors.

If the tube A is quadruple the tube B, with which it has a communication by the channel C, and any liquor be put into it, it will rise to the same height in both tubes.

Demonstrat. If there be 4 pounds weight of liquor in the tube A, there will be 1 pound in the tube B, since both tubes will be equally filled. But the 4 pounds in the tube A cannot sink 1 inch, for example, but the pound in the tube B must mount 4 inches: Therefore the ratio of the force A, to the force B, being reciprocally as the ratio of the velocity of B, to the velocity of A, the weight A cannot at all raise the weight B, (by *Axiom.*)

2. If you put into the tube B, a different liquor from that in the tube A, it will still remain in equilibrio, since the liquor in the tube B will weigh as much as that in the tube A, if it rise in the tube B to the same height as in the tube A: For example, if into the place of the pound of liquor in the tube B, a pound of other liquor be poured, it will make an equilibrium with the 4 pounds in the tube A, to whatsoever height it rises in the tube B.

COROLL

COROLL. I.

If the body A be put into water, it will so far sink as till it takes up a space equal to a mass of water of its own weight.

Demonstrat. The body A, with the water under it, forms a cylinder, making an equilibrium with all the rest of the water, as with another cylinder: Therefore (by *Cor. preced.*) this cylinder composed of the body A, and the water under it, ought not to weigh more than if it was all water, and did not rise above the surface of the water; therefore the body A weighs as much as the mass of water, whose place it occupies under the surface of the water. *Fig. 14.*

COROLL. II.

In measuring the capacity of that part of a ship within the water we find its burden: For if, for example, we find the part of a ship which is under water to be equal to 1000 cubick feet, we conclude, that the ship, with all its lading, &c. is equal in weight to 1000 cubick feet of water.

H S

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P R O P. IX.

Of suspended Bodies.

L E M M A I.

If from any point, as G, of the line BG, the arch BD be described, and the right
Fig. 15. lines BD, BA, be drawn, and DF fall perpendicularly upon BA; lastly, if AI be parallel to BD, I say, 1. That when the point F is between the points A and B, BD will have a less ratio to BF, than BA to AI.

Demonst. The angle BDF being the complement of DBF, will be greater than the angle ABI, which together with the angle DBF makes but an acute angle. Imagining therefore the angle LDB equal to the angle ABI and the alternate interior angles BAI, ABD, being equal, (by 15. 1.) the triangles DBL, BAI, will be similar; and consequently (by 4. 6.) $BA : AI :: DB : BL$; but BD is less with regard to BF, than to BL, (by 8. 5.) therefore BD is less with respect to BF, than BA to AI.

2. If the point B fall between the point F, and
Fig. 16. the point A, BD will have a greater ratio to BF, than BA to AI.

Demonst. The angle BDF being but the complement of DBF, or ABL, will be less than the angle ABI, which, with ABL, makes an obtuse angle; for GBD is acute, (by 16. 3.) therefore the rest will follow, as above.

L E M M A

L E M M A II.

If instead of BD, another line be drawn, as BC, and from the point C, the arch ED be described, and AH drawn parallel to BC, I say, 1. That when the point F falls between the points A and B, EB will have a less ratio to BF, than BA to AH. Draw the line EF. *Fig. 15.*

Demonst. Since the alternate angles EBF, BAH, are equal, it only remains to prove, that ABH is less than BEF, (by preced.) which is evident : for if thro' the three points D, E, B, a circle be described, the point F will be found within it, since the angle BED is obtuse, and BFD right ; therefore the angle FEB being subtended by a greater arch than BDF, will be greater, (by 22. 3.) but the angle BDF is greater than ABH, because it is the complement of the angle ABD, which, with ABH, makes but an acute angle ; therefore the angle BEF will be greater than ABH.

2. If the point B falls between the point A and the point F, BE will have a greater ratio to BF, than BA to AH, because *Fig. 16.* the angle BEF is less than BDF.

Demonst. If a circle be described thro' the three points B, D, F, the point E will fall without the circle; because the angle BFD is right, and the angle BED acute ; therefore the angle BEF will be less than BDF, which being but the complement of DBF, will be less than ABL.

IF

If the weight B is suspended by a cord, one end of which, as H, is fixed, and the other end D, passing thro' the pully K, sustains the weight D; I say, that when the weight B, shall be to the weight D, as the line of direction AB to AL, parallel to the line BN, that then the weights shall be in equilibrium.

Demonstrat. As the weight D cannot raise the weight B, to the point G, (by *Ax.*) because its velocity BG, bears a less proportion to the velocity BE, of the weight B, (by *Lem.* 1. & 2.) than the force of the weight B, to the force of the weight D, so neither can the weight B, descend to the point I, (by *Ax.*) because the velocity BF, of the weight D, bears a greater proportion to the velocity BR, of the weight B, than the force of the weight B, has to the force of the weight D, (by *Lem.* 1. & 2.) and consequently they will remain in equilibrium, (by *Ax.*)

If the point H be not fix'd, but the cord passing thro' the pully sustains the weight C, it will still be the same thing; for the weight C will only serve to shorten the cord HB; so that we may conclude, that if the three weights D, B, C, are in equilibrium, the weight B will be to the weight D, as BA to AL, and to the weight C, as BA to AN, which is supposed parallel to HL; and consequently the weight D, will be to the weight C, as AL to AN, or as the sine of the angle ABL, to the sine of the angle ABN.

PROP. X.

Of Acceleration.

Acceleration is the augmentation or increase of motion, caused by a continual application of the power.

	B			A			C		
O.	1.	2.	3.	4.	5.	6.	7.		
7.	7.	7.	7.	7.	7.	7.	7.	7.	

LEMMA.

The sum of the terms of an arithmetical progression, beginning with 0, is equal to half the sum of an equal number of terms, each being equal to the last term of the progression.

Demonst. If you take two terms of the progression, of which the one A, is as far distant from the last C, as the other B, from the first 0, their sum will be equal but to the last term C; because the term C exceeds the term A, just as the term B exceeds 0: Thus taking all the terms of the progressions two and two, each sum will be equal to the last term C; therefore since the number of these sums will be but half the number of the terms, consequently the sum of all the terms of the progression will be but half the sum of the same number of terms, each equal to the last term C.

1. If the weight A, falling in a non-resisting medium, runs thro' the space AB, during the first minute of its fall; during the second, it will run thro' the space BC, which is triple the space AB.

Fig. 18.

Demonst.

Demonst. Experience teaches us, that gravity continually increases the motion of heavy bodies, in their descent towards the earth. Thus imagining the first minute divided into an infinite number of parts, we may conceive the motion of the weight A equally augmented thro' each of those parts; and that having had no motion in the beginning of the minute, during the second part of the time; it will have run thro' one part of the space AB; than 2 in the third, 3 in the fourth, &c. We may further conceive, that throughout all the parts of the second minute, it will have the same motion it had at the end of the first; and consequently, if the weight A received no new motion during the second minute, it would run thro' a space double the space AB; then since it does acquire a new motion, equal to that it got during the first minute, it will run thro' a space triple to the space AB.

2. In the same manner it may be proved, that during the third minute it will run thro' the space CD, which is five times the space AB; from whence we conclude, that the spaces run thro' will be in a duplicate ratio to the times of the descents, or as their squares.

3. The impression made by one body upon another, is greater or less according to the motion; that is, (according to *Def. 3.*) with regard either to the bulk of the body, or its velocity.

COROLL.

COROLL. I.

If the weight A in its fall strike a body at the point C, the impression it will there make upon such body, will be quadruple what it would have been at the point B. Thus the impressions made by a body falling, increase in a duplicate ratio of the times of descent, or as the spaces run through. *Fig. 18.*

COROLL. II.

If in the room of a dead weight, we substitute a living power, as a man's hand A, striking a mallet against the wedge B, *Fig. 19.* the same reasoning will hold good, that is, the impression of the mallet A on the wedge B, will be as the space AB.

COROLL. III.

It is not altogether the same thing with respect to springs, because their force decreases in proportion as they approach their natural situation. Thus the impression made by the string of a bow upon an arrow B, will not increase as the spaces BB. *Fig. 20.*

COROLL. IV.

Two bodies equal in every respect falling by the force of gravity from the same height, will make the same impression on the body they strike; or if two unequal bodies fall from different heights, if those heights are reciprocally as the squares

squares of the weights of the descending bodies, their impressions will be likewise equal.

COROLL. V.

One body simply laid upon another, will make the same impression on it, as would another body falling from above, provided the acquired velocity of the one is to the simple motion of the other, reciprocally as the gravity of the first to the gravity of the second. But as we know not the simple motion of heavy bodies, that is to say, the motion they would have if they were to descend without acceleration, we can determine nothing upon that point.

4. If a lever AC, without gravity, were so fastened at the point A, as that it might
Fig. 21. turn upon it, and being so raised as to form any angle with the horizontal line AB; if then a weight had been placed upon a point D, the lever will fall towards the horizontal line AB, with less velocity, than if the same weight had been applied at the point C, because the lever in both cases is supposed void of gravity.

COROLLARY.

If the weight be placed at the point D, it will not fall with so much force as if it were placed at the point C; nor will the impression it makes upon the body upon which it falls be so great; and therefore it is, that the longer the handle of a hammer is, the greater is the force of the blow.

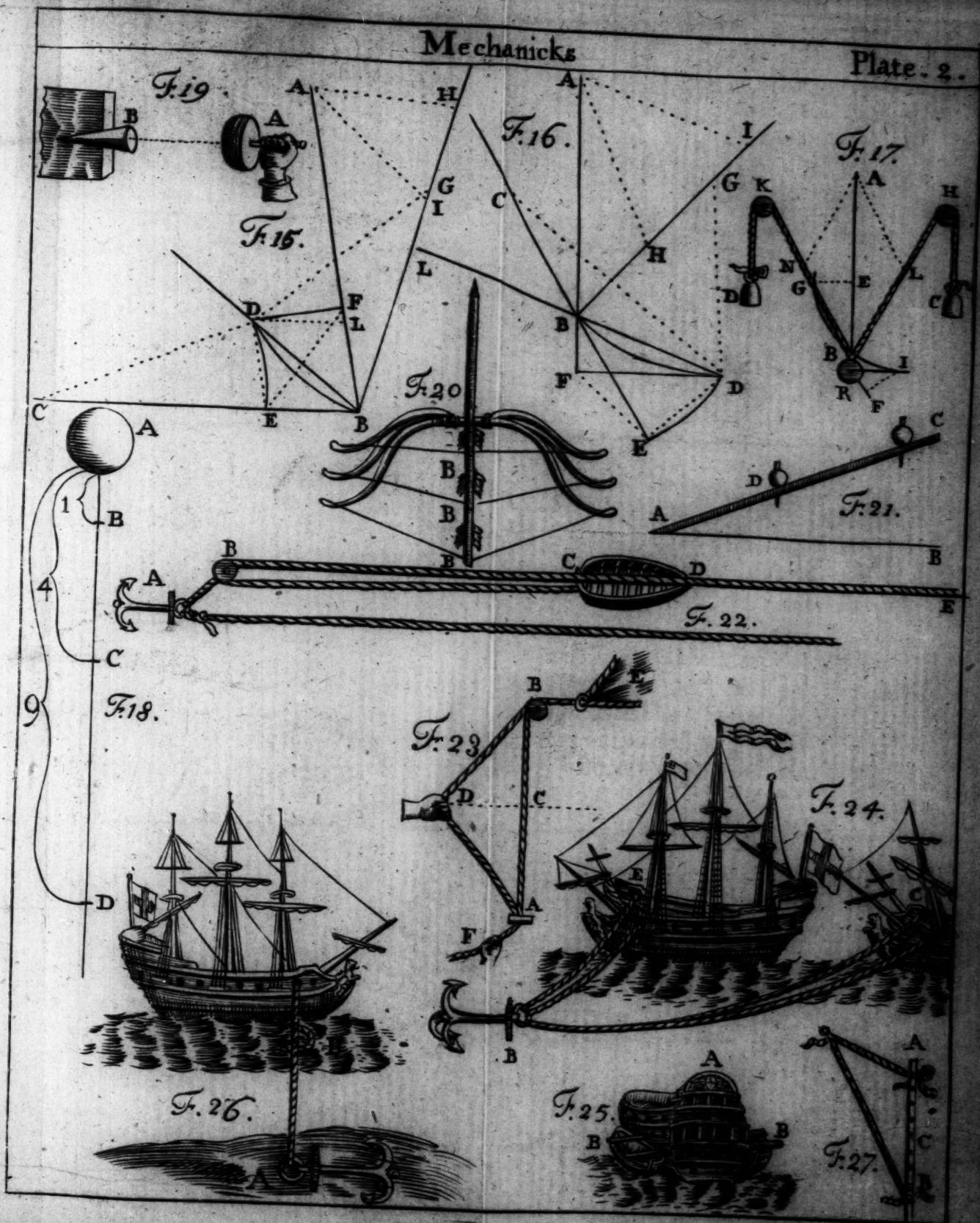
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PROP. XI.

The Application of these Principles to some Problems useful at Sea.

1. With regard to Acceleration.

Suppose the anchor A fixed in the bottom of the sea ; to the end of the buoy-rope AB, fasten a block B, thro' which *Fig. 22.* rieve a rope, fasten'd at one end to the stern of the boat CD, so that the other end passing thro' the blocks C and D, (fixed in the stern and stem of the boat) may be brought aboard the ship, which is likewise supposed to have an anchor out a-stern : It will then be easy to imagine, that when that part of the rope on board, is haul'd with due force, the boat will be drawn towards the anchor ; and as the force of hauling is continued, the motion of the boat will be augmented ; when therefore the boat comes a-pique the anchor, not being able to approach it nearer, it will draw the anchor with such force as to raise it ; and the cable being recovered, it may be brought aboard.

COROLLARY.

By this means an anchor may be weigh'd, when by reason of the enemies fire, a boat cannot be mann'd out with safety to recover it ; it being in this case sufficient only to send out a driver to fix the block, and rieve the rope.

2. With

2. *With respect to suspended Bodies.*

1. Suppose the rope or tack AB, of which the end A, after passing round a block A, *Fig. 23.* is extended to the power F, which holds it, and the other end B, passing likewise thro' the block B, is fastened to the clue of the sail E: Laying then hold of the rope at the point C, it may be easily drawn towards D, till the resistance of the sail is to the force of the power D, as the sine of the angle CAD to the sine of the angle CDA, (by *Prop. 9.*) without obliging the power F to give way, since the block A, against which the rope AB bears hard, being immoveable, will not let it run. When therefore the power D returns to the point C, the power F will recover part of the rope ADB, especially if the power D opposes itself to the power E, in drawing the rope towards A. Thus after several attempts the tack may be belayed as it ought.

2. Suppose the cable AB, one end of which is fixed in the vessel A, and the other *Fig. 24.* fastened to the anchor B; suppose another shorter cable BD: It is evident, that these two cables being suspended between the two points to which they are fixed, the cable AB will bear hardest upon both points, and also with respect to itself, as being heaviest: Notwithstanding which, the anchor B will strain less upon the cable AB, than upon the cable BD; and the cable AB will be less subject to break, or part, as the seamen phrase it; 1. Because the cable AB being longer, will form a more concave figure, (by *Prop. 9.*) great part thereof will rest upon the ground

ground near the anchor; upon which therefore it will have less strain, and particularly will not raise up its ring, as the cable BD, which renders it more liable to break its hold. 2. Because when the swell of the sea raises the ship's head, as from the point A to the point C, or from the point D to the point E, its effect is plainly greater upon the cable BD, than upon the cable BA.

3. *With respect to Lighters.*

1. If an unladen ship A, be aground in a haven, she may easily be floated, by placing on each side a lighter or barge so loaden *Fig. 25.* that their gunnels be brought down almost to the surface of the water, then fixing beams in the ports of the ship, let them rest upon the sides of the lighters, extending quite over on both sides, then lightening the lighters, as they rise, they will lift up the ship.

2. The same thing may be done with large vessels fill'd with water, which being fastened to the sides of the ship, the water being pump'd out will in like manner raise the ship, as the vessels begin to float upon the water.

4. *With respect to Cranes and Pullies.*

The use of Cranes and pullies is so common, that I shall not need dwell much upon it; I shall therefore just observe, that what obstructs the force of capstans when the ship or boat is brought a-pique the anchor, not only proceeds from the resistance of the anchor itself before it quits its hold, but also from the rubbing of the cable

cable in the hawse. When therefore the anchor's hold seems too great for the power of the capstan, fix a rope (or messenger, as the seamen term it) upon some part of the cable, as B, and after having brought a large tackle upon it, bring the fall of the tackle to the capstan, which then will raise the anchor without difficulty, not only as the tackle will have a better purchase, and will consequently augment the force, but because the rubbing of the cable in the hawse will likewise be thereby prevented.

R E M A R K.

Sometimes an anchor is so fixed or buried in the ground, that nothing is able to raise it. In such case, if the tides are considerable, they let the ship ride with her cable as much as possible a-pique during the ebb, and then as the ship rises with the flood, either the anchor is raised, or the cable breaks.

5. *With respect to Levers.*

To raise a yard or mast, as AB, to place it in a ship, it is most convenient to fasten the purchase at the end A; for if it be seized by the middle C, the resistance will be found twice as great, (by *Prop. 2.*) and if it be seized about the end B, which is still farther distant from the place it is to be drawn to, a still greater force must be applied.





FORTIFICATIONS.

 *O fortify* a place, is to put it into a condition of being defended by a few men against the attack of a much greater number. As the several rules laid down in this Tract are all conformable with this definition, so the following Axioms may be look'd upon as immediate consequences thereof.

A X I O M S.

1. The *Fortifications* of a place ought to be as little extended as possible.

2. They ought to be as uniform as possible.

3. They ought to be so contrived as at once to cover the defendants, and discover the approaches of the enemy, and adapted to the common weapons of defence.

4. They ought, as much as possible, to be such as are easy to build, hard to destroy, and not difficult to be repaired.

5. Each

5. Each part should be defended by as many places as may be, and liable to be attacked from as few places as possible.

DEFINITIONS.

Fig. 1. 1. The *Rampart* is a bank of earth encompassing the place within the walls, as EBE.

2. A *Bastion* is an advanc'd part of the rampart, in form of a Pentagon, as B.

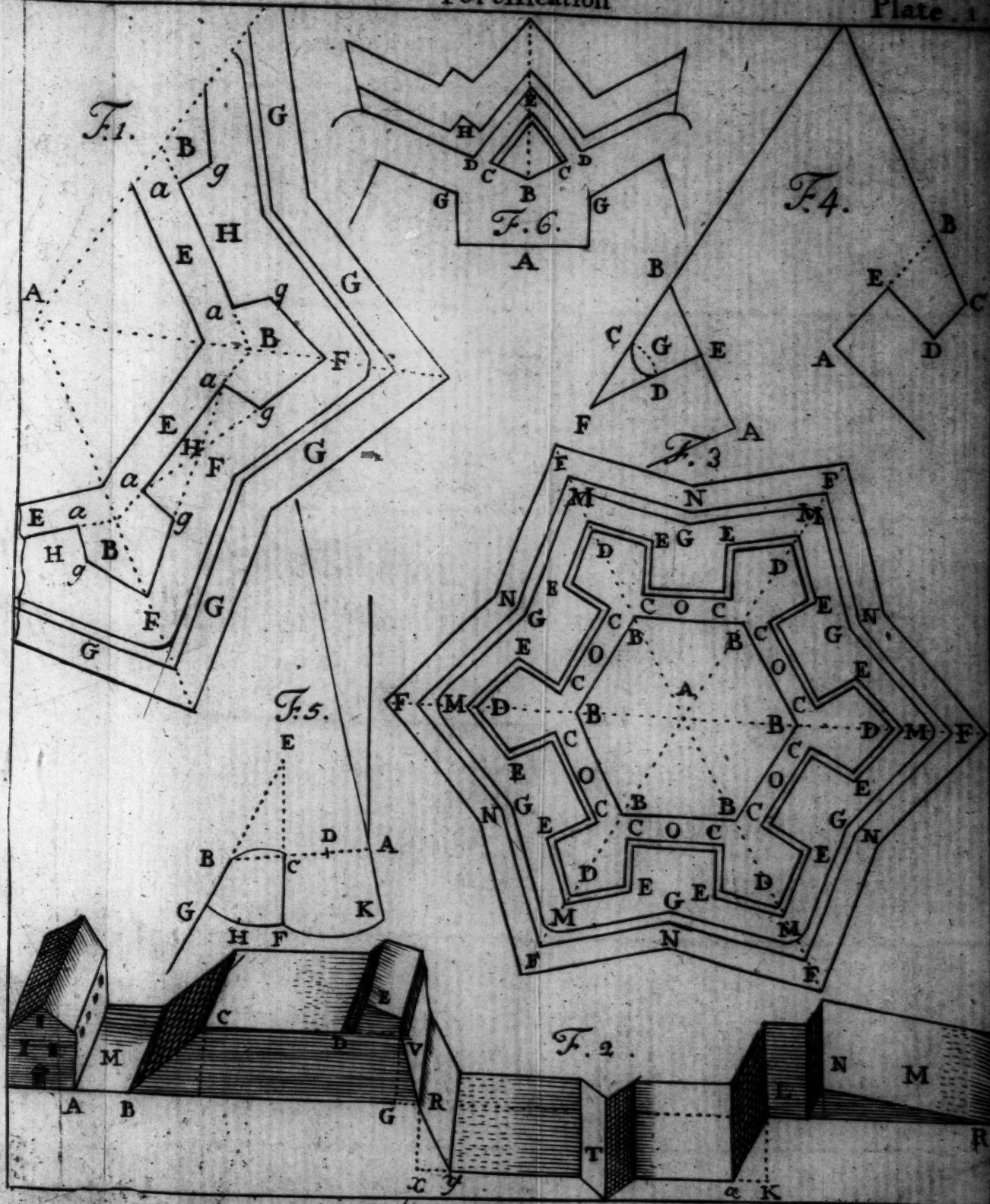
3. The *Curtain* is that part of the rampart lying between two bastions, as E.

4. The *Moat* is a large ditch encompassing the place without the walls, usually fill'd with water, as HF, HF.

5. The *Cover'd Way*, FFF, is a passage along the border of the ditch opposite to the place, and is covered with a bank of earth falling slopeways towards the country, till it comes to the level of the ground. This slope is call'd the *Glacis*, as G, G, G.

6. The *Profile* of a Fortification is the section made by a plane cutting it from the middle A, to the extremity of the glacis K. There are commonly two made, one passing thro' the middle of the curtain, the other thro' the middle of the bastion.

7. In



7. In the Profile AR, supposed to pass thro' the middle of the curtain, there are to be considered, 1. The *Place of arms* Fig. 2. M, which is a street or void space between the houses and the rampart. 2. The *Talus* or Slope of the rampart BC. 3. The *Rampart* CD. 4. The *Banquet* or little bank D. 5. The upper surface, of which E is call'd the *Parapet*. 6. The *Sone face* or front of the wall V. 7. The *Scarpe*, or steep bank of the ditch next the town VR. 8. The *Moat* or ditch RL. 9. The *Channel* of the moat T. 10. The *Counterscarp*, or that bank of the ditch opposite to the place L. 11. The *Covered Way* L. 12. The *Palisade* N. 13. The *Glacis* of the counterscarp M.

8. The lines F g being prolonged to the curtain, are called *lines of defence*; and the angle g H g, which they form, is called Fig. 1. the *flanking angle*. The planes F g of the bastion are called its *faces*; the planes a g its *flanks*; and the planes a B its *demigorges*. The angle of the faces g F g is the *flanked angle*; the angle g a a, is the *angle of the flank*; the angle a g F, the *angle of the epaul* or *shoulder*; the angle a B a, the *angle of the gorge*; the angle BAB, the *angle of the center*.

PROP. I.

The manner of laying down upon paper the Plan of a regular Fortification.

Regular Fortifications are such as are made upon a regular Polygon.

1. Measure

1. Measure exactly the circuit of the place to be fortified, at about 12 paces distant from the houses, and divide the whole circuit by 150 geometrical paces at least, or by 180 at most, and the quotient will give the number of bastions, in such manner that their lines of defence shall not exceed the carriage of a musquet.

2. Inscribe in a circle, a polygon with as many sides as the place is to have bastions.

Fig. 3. and from the center A, thro' the angle B of the polygon, draw lines at pleasure, which lines are called *Principals*. Afterwards take Bc equal to $\frac{1}{3}$ of the side of the polygon, and BD equal to $\frac{1}{8}$, then draw the lines of defence Dc, and from each point c, raise perpendiculars, which meeting the lines of defence in the points E, will form the bastions c E D E c.

This method seems preferable to all others, as it is the most simple, and makes the bastions well proportioned: It avoids forming the flanked angles too acute, or too obtuse, and gives the flank all the extent it is capable of.

3. Having thus described the outward circuit of the rampart, draw from the angles of the shoulder E the lines EM, parallel to the faces DE, which will meet each other in the principal lines at the points M, and before the curtains in the points G, and so will determine the outward circuit of the moat M, M, M, which ought to be rounded before the angles D, by an arch described from the point D, having for its radius the perpendiculars Dy.

4. To

4. To finish the plan, draw within the place, lines parallel to those which form the outward circuit of the rampart : 1. At the distance of $\frac{1}{3}$ of the flank, for the parapets. 2. At the distance of $\frac{1}{2}$ the demigorge, for the ramparts. 3. At the distance of 5 paces from the rampart, for the place of arms. 4. At the distance of 16 foot within the rampart, for its talus or slope. 5. At the distance of 5 foot, from the parapet, for the banquet. In the same manner, on the outside of the moat, must be drawn lines parallel to its outward circuit : 1. At the distance of $\frac{1}{4}$ of the flank, for the cover'd way. 2. At the distance of $\frac{2}{3}$ of the flank, for the glacis.

5. It will be easy to lay down the profile, and to assign the several heights : For sup-
 posing AR to represent the level of the *Fig. 2.*
 plane ; take AB, 5 paces, for the place of
 arms BA, and the perpendicular OC, 16 foot, for
 the talus or slope BC ; the thickness of the lower
 part of the rampart BR, 12 paces, the upper part
 CD to the banquet, 6 paces $\frac{2}{3}$; the banquet 6 foot ;
 the thickness of the lower part of the parapet DV,
 3 paces $\frac{1}{3}$; the upper part 2 paces $\frac{1}{3}$; its inward
 height ED 6 foot ; its outward height EV, 5
 foot ; the talus of the rampart GR to the moat, 7
 foot ; and the talus of the scarp xy, 2 paces ; the
 depth of the moat x R, let be 16 foot ; the width
 of its channel T, 15 foot ; the talus of the coun-
 ter-scarp a K, 10 foot ; the banquet of the cover'd
 way, 5 foot.

I

REMARKS.

4. To

REMARKS.

I.

Formerly it was an usual practice to raise at the foot of the scarp a bank of 30 or 40 foot broad, shelter'd with a parapet of 15 foot thick, and 6 foot high, which was call'd the *fausse-braie*; but as such place is too much exposed to be of any great service, it is not now used.

II.

As the flanks are the grand defence of fortifications, they are sometimes cover'd with an *oreillon* or an *epaulement*, after different manners.

1. Divide the flank BA equally in the point E, and raise the perpendicular ED $\frac{1}{3}$ of the flank; then draw DC parallel to the flank, till it meet the face prolong'd in the point C, so shall you have the *epaulement* BCDE. But if you would have an *oreillon*, from the point F, where the line ED meets with the face, with the distance FC, describe the arch DC; and from the middle of that arch G, with the distance GC, describe an arch which will form the *oreillon* BCDE.

2. Mr. *Vauban's* flanks may likewise be very nearly imitated, thus: Take BC $\frac{1}{3}$ of the flank, and from the point C draw FC equal to BC, in such manner that being prolong'd it will meet the flank'd angle of the next bastion: Farther from the point E, where the face prolonged meets with it, with the distance EF describe an arch FG, the middle of which

which H, will give you the center of the arch BC. Lastly, from the point D, the middle of the line CA, with the distance DF, describe the arch FK, which will meet the line of defence in the point K, and draw KA, which will finish the flank AKFCBG.

III.

I say nothing of *Casemates*, which are now in a manner out of use.

P R O P. II.

To lay down Outworks upon paper.

Under the name of *Outworks* is here comprehended every kind of work detached or separated from the rampart of the place, advancing farther towards the campaign.

1. To raise a *ravelin* before the curtain: From the middle of the curtain A, draw an indeterminate perpendicular, upon which *Fig. 6.* set off AE equal to the curtain, and from the point E draw lines to the angles of the shoulder, which cutting the outward bank of the moat in the points C, will give the outward circuit of the ravelin ECBCE. As to the thickness of its parapet, its cover'd way, and its glacis, they take their proportions from those of the place itself; but the width and depth of the ditch are but of half the dimensions allow'd to the grand moat of the place; which observation is general with respect to all sorts of outworks.

2. To erect an *half-moon* before the flank'd angle,

Take upon the principal line from the point *o*, where it cuts the outside of the moat, *o B*, equal to $\frac{2}{3}$ of the face; then having drawn, from the point *B*, lines to the flanking angle *G*, produce the faces *FA*, till they cut the bank of the moat in the points *D*, and the lines *BG* in the point *E*, and you will have the exterior circuit of the half-moon *o D E B E D*.

3. For *Horn-works*: From the middle of the curtain *A*, draw the perpendicular *AB*,

equal to the side of the polygon *NN*,

and from the angles of the flank *G*, draw *GC* parallels, and equal to the line *AB*, they will cut the outward bank of the moat in the points *D*; then take *BE*

equal to $\frac{1}{4}$ of the curtain, and draw *CE*, which will finish the horn-work *DCECD*.

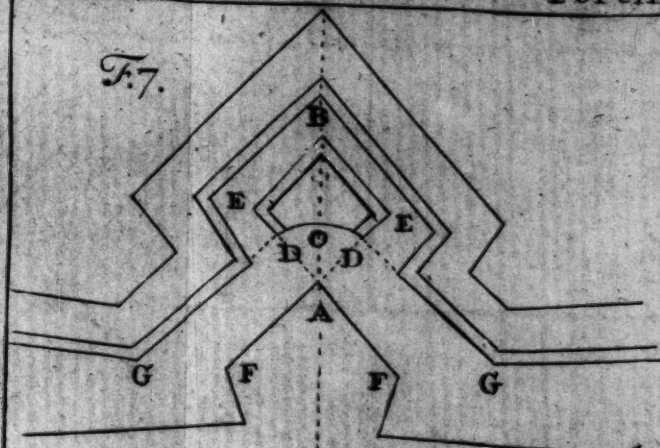
Or if you like it better, taking *CE* $\frac{1}{3}$ of *CC*, and *CF* $\frac{1}{4}$ of *CC*, draw the perpendiculars *EH* upon the line *FF*, and they will cut

the lines of defence *CH* in the points *O*, and so will give the circuit of the horn-work *DFCOHHOCFD*.

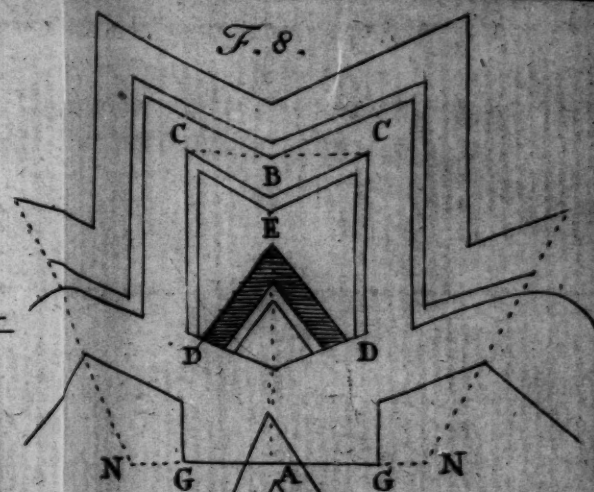
Or lastly, from the point *B*, drawing the perpendiculars *BC* equal to half the curtain, the lines *AC* will cut the outward bank of the moat of the half-moon in the points *D*; then having taken *BO*, and *BE*, $\frac{1}{4}$ of *CC*, and having drawn the lines *CE*, from the middle of them *G*, draw *GO*, and you will have the outward circuit of a swallow's tail *CDGOGDC*.

4. For

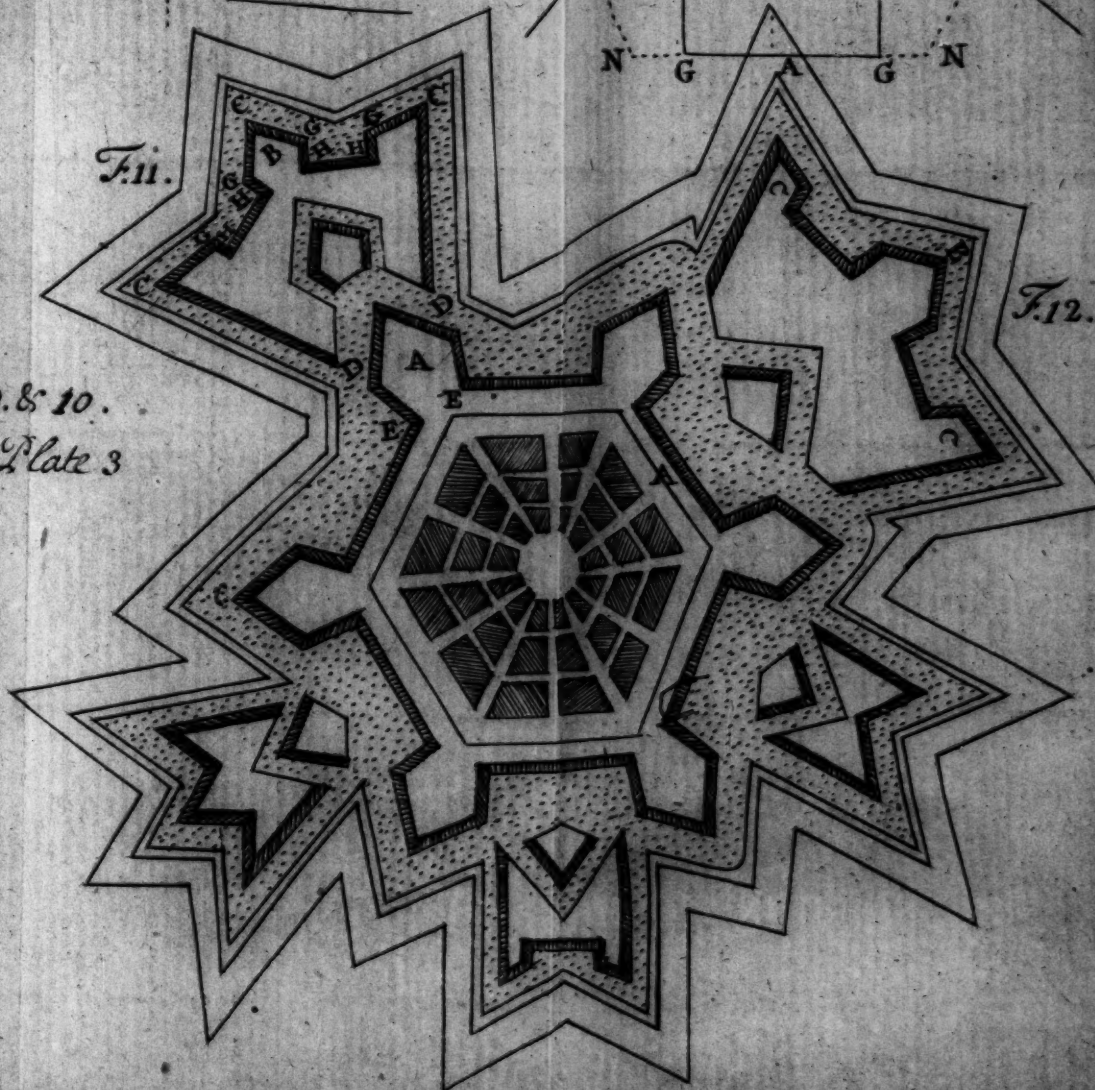
F.7.



F.8.

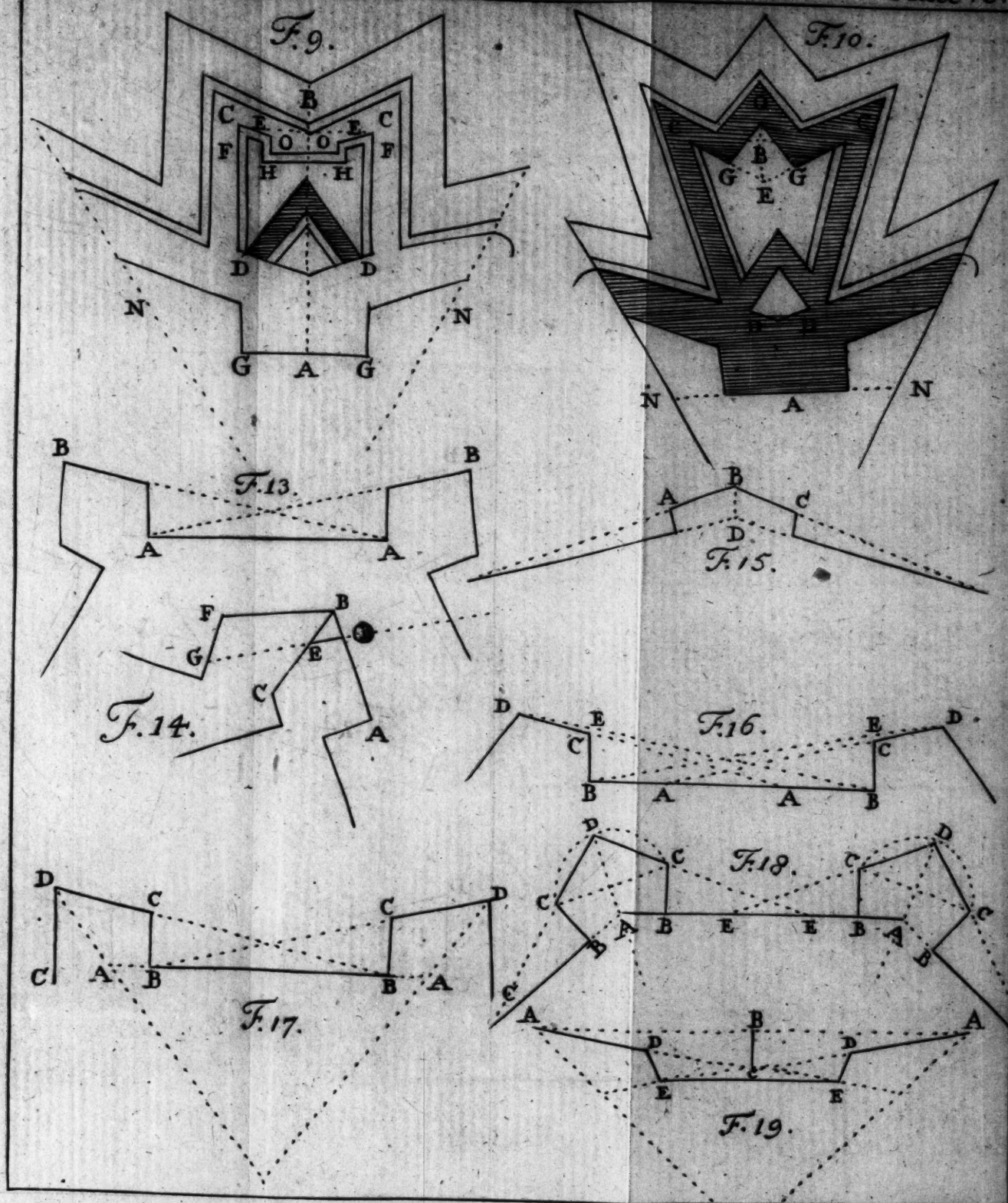


F.11.



F.12.

Fig. 9. & 10.
Vide Plate 3



4. For *Crown-works* : 1. From the angle of the polygon A, upon the principal line take AB, equal to $\frac{2}{3}$ the side of the polygon; then making BC on both sides equal, and parallel to the sides of the polygon, draw from the angles of the flank E the lines EC, which will cut the outward bank of the moat in the points D. Farther, make two demi-bastions upon the lines, on each side one, after the manner laid down in the second manner of describing a horn-work, (*vide* Fig. 9.) and you will have the exterior circuit of the crown-work DCGH HGBGHHGCD. *Fig. 11.*

2. The same thing may be done in drawing AB from the middle of the curtain, and making the angle CBC equal to the angle of the polygon. *Fig. 12.*

PROP. III.

The Demonstration of the foregoing Rules.

To demonstrate the foregoing rules, let them be applied to several polygons, and it will appear that they will always produce Fortifications conformable to the axioms at first laid down, more especially the following faults will constantly be avoided.

r. It is a great mistake in Fortifications, when the lines of defence AB are too long, because the enemy may then with safety approach the point B, where they will be in security, at least, from musket-shot, till they have

have time to shelter themselves. Now, by the foregoing rules, the greatest length of the said lines of defence is but 180 paces.

2. When the flanked angle ABC is too acute, the enemy's cannon will easily make a
Fig. 14. breach; for the point D being only sustain'd by the small thickness DE , the ball will quickly shake it; whereas in a bastion less acute, as ABF , the point D is sustain'd with the whole thickness DG . Our method therefore gives the flank'd angle as little acute as possible, nay, very often obtuse.

3. It is true, the flank'd angle ought not to be so obtuse as to make the bastion too
Fig. 15, straight, as ABC ; but in other respects they are the more substantial, and better, provided the said inconvenience be avoided, as we have done in fixing the length of the capital BD .

4. The main defence being in the flanks, they ought not to be too short; one method makes them longer than commonly they are laid down, except in the square, where large flanks would render the flank'd angle too acute.

5. It is true, our method excludes second flanks AB , so much recommended by some
Fig. 16. Dutch Engineers, that the face of the bastion CD may not only be defended by the fire from the flank BC , but likewise from the second flank AB . Nevertheless, it appears to me, that the defence from the second flank, where the guns must be pointed slopewise over a
 very

very thick parapet, cannot be very considerable; and I should rather chuse to enlarge the flank BC, by the length CE, than to have recourse to a second flank AB.

PROP. IV.

Some other Methods.

I.

Some Engineers (as *De Ville*) divide the side AA of the polygon into 6 equal parts, and make the demigorge AB one of *Fig. 17.* those parts, as also the flank BC; then for places which have less than 6 bastions, they draw the line of defence DCE, which gives the flanked angle CDC. For *Fig. 18.* other places they describe the semicircle upon the line CC, which cutting the principal in the point D, gives the flank d angle CDC. This method, in my opinion, makes the bastion too small, nor can I see why they should prefer right angles to obtuse ones, unless it be for the sake of a second flank, which I have before observed to be of little advantage.

II.

Others (as *Pagan*) begin with the exterior side of the polygon AA, which they divide equally in the point B; then *Fig. 19.* they draw the perpendicular BC, which they extend to 30 fathoms, and after having drawn the indeterminate lines ACE, they set off AD 30 fathoms, and from the point D, they draw the flanks DE perpendicular upon the lines of defence EA; and lastly, they draw the cur-

tain EE. They think, that the flanks being perpendicular upon the lines of defence, they are most advantageously placed, their fire being thereby render'd more large, and more direct: Nevertheless, I cannot prefer these sort of flanks to those commonly used, because they are more exposed to the enemies batteries, without any possibility of shelter.

III.

There are some Engineers who give $\frac{1}{4}$ of the side of the polygon to the demigorge; but by this means the flank, and consequently the defence, becomes much lessen'd: However I do not reject this manner in polygons above the decagon.

IV.

Others, to make amends for what is wanting in flank by the foregoing method,
Fig. 20. make a flank in the middle of the face, and sometimes two; but the face is
Fig. 21. thereby rendered much the weaker.

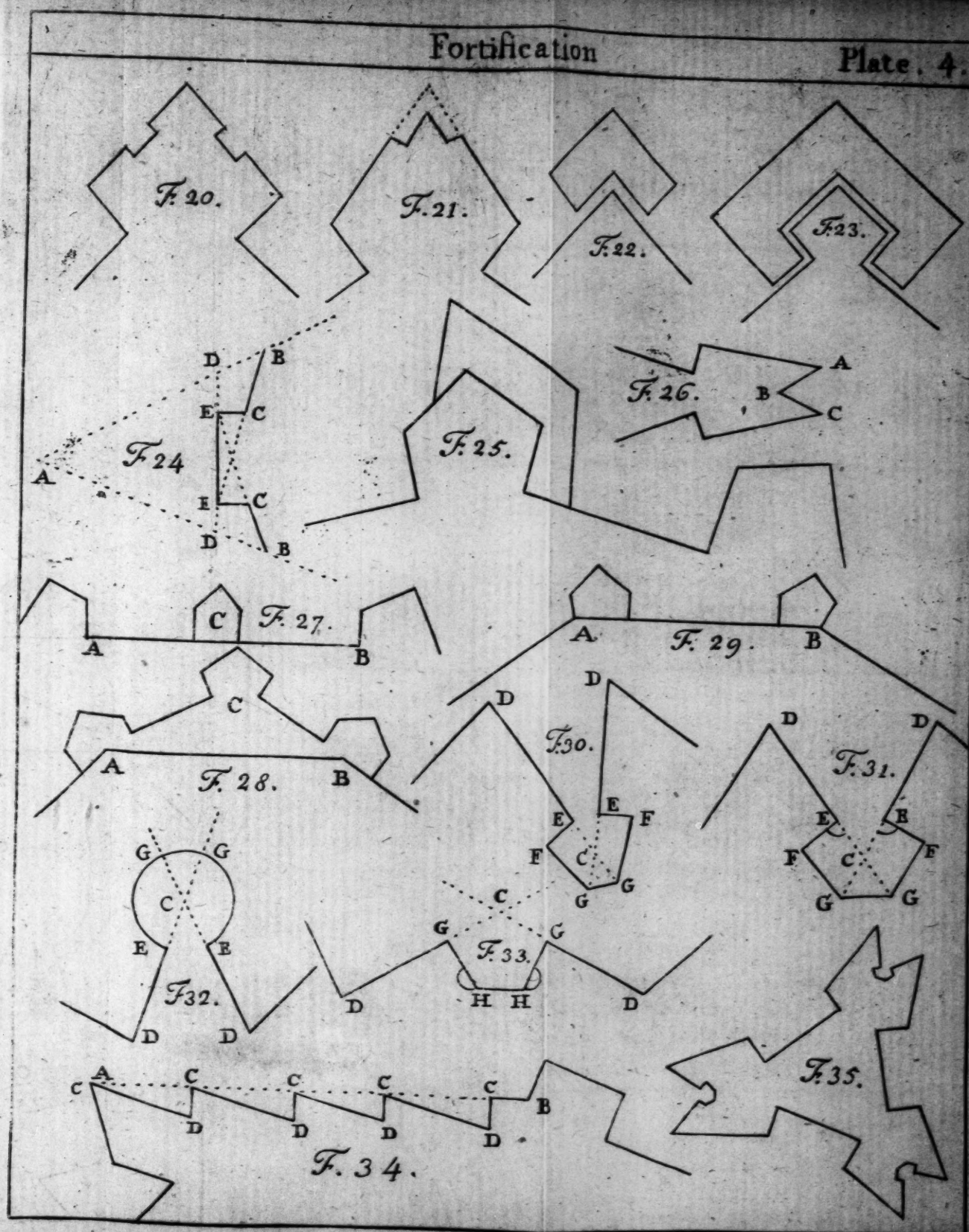
V.

There are also some who would have the bastions detached from the place, and
Fig. 22. separated by a ditch, the more easily to obstruct the progress of the enemy after he has taken the bastion.

VI.

Others are for making the bastions of the place extremely small, and then surrounding
Fig. 23. them with other detach'd bastions, which ought nevertheless to be commanded by the first.

P R O P.



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PROP. V.

To compute or calculate Fortifications.

It is of great importance, after having traced out Fortifications upon paper, to bring them to a computation, that is, to find the length of all the lines and the value of all the angles, which will afford much help to the laying them down afterwards upon the ground.

1. For the greater exactness, we begin by multiplying the side DD of the polygon by 100; then cutting off the two last figures of every number found, the two figures so cut off, with the denominator 100, will form a fraction. *Fig. 24.*

2. In the triangle ADD the angle A may be known, by dividing 360 by the number of sides in the polygon; then dividing the supplement of the angle A into two equal parts, you will find the two angles D, which are equal; after which say, (by *Trig. 13.*) As the sine of the angle A: to the side DD :: so is the sine of the angle D: to the side AD.

3. In the triangle DEB, the angle D being known, (by the preceding, as also the side BE, which is $\frac{4}{5}$ of the side DD, (by construct.) and the side DB, which is $\frac{1}{3}$ of it, the whole triangle DEB may be resolved, (by *Trig. 13.*) by which means will be found both the angle B, which is $\frac{1}{2}$ the flanked angle, and the line of defence BE, as also the angle E.

4. In the triangle *ECE*, rectangular at the point *E* of the line *CE*, (by construct.) the side *EE* being known to be $\frac{1}{3}$ of *DD*, and the angle *E* to be right; you may resolve the triangle *ECE* (by *Trig.* 12.) by which you will know the flank *CE*, and the line *CE* opposite to the right angle, which being subtracted from the line *BE*, will give the face *BC*: You will likewise by the same means know the angle of the shoulder *ECB*.

5. The foregoing rules may also be easily applied to outworks. The following calculation is an example of an heptagon, whose side is supposed equal to 170 fathoms.

The computation of an Heptagon.

<i>Lines.</i>	<i>Fathoms</i>	<i>Angles.</i>	<i>Deg. Min.</i>
Inward side —	170	Angle of the center }	51 25
Radius — —	196	Angle of the Polygon, or }	128 34
Demigorge —	34	of the gorge }	
Capital — —	57	The flanked angle }	93 36
Curtain — —	102	The flanking angle }	145 2
Line of defence —	168	The angle of the flank }	107 28
Flank — —	32		
Face — —	61		
Outward side —	218		

P R O P.

PROP. VI.

To trace out irregular Fortifications upon paper.

Irregular Fortifications are such as are raised about irregular polygons: But in these irregular works, we must follow, as much as possible, the proportions laid down for the regular ones.

S E C T. I.

Of irregular Bastions.

1. As Bastions ought to be so built that their cannon may sweep the surface of the circumjacent campaign, if they are found too high for that purpose, let a second bastion be added beneath the first, or even a third, if it *Fig. 25.* is necessary, still taking care that their faces be well defended. But if a bastion is too low, raise a cavalier upon it. A cavalier is a mass of earth, well beat, of about 26 fathoms radius, whose center is commonly placed in that point of the bastion where the defences meet; it is commonly made two fathoms high, and with a talus of 7 foot; it ought also to have a parapet of 3 paces broad, and 6 foot high; it may be made of any figure, but most commonly is formed as the bastion.

It will sometimes happen, that the faces of a bastion would become excessive long if they were to be extended till they meet; *Fig. 26.* in such case, they are usually closed with a returning angle, the manner of fortifying which shall be hereafter laid down.

3. If

3. If one side of the polygon, as AB, is too long, it may be remedied several ways : 1. If it be long enough to admit a bastion in the middle, let one be raised, as C. 2. If it is not long enough to admit such bastion, it will be proper to form a falliant angle, as A C B, 3. Or the bastions on each side may be advanced nearer each other, and both intirely form'd upon the line AB.
- Fig. 27.* is too long, it may be remedied several ways : 1. If it be long enough to admit a bastion in the middle, let one be raised, as C. 2. If it is not long enough to admit such bastion, it will be proper to form a falliant angle, as A C B, 3. Or the bastions on each side may be advanced nearer each other, and both intirely form'd upon the line AB.
- Fig. 28.* a bastion in the middle, let one be raised, as C. 2. If it is not long enough to admit such bastion, it will be proper to form a falliant angle, as A C B, 3. Or the bastions on each side may be advanced nearer each other, and both intirely form'd upon the line AB.
- Fig. 29.* bastions on each side may be advanced nearer each other, and both intirely form'd upon the line AB.

S E C T. II.

Of returning Angles.

They ought to be avoided as much as possible ; but if the ground obliges you to have recourse to them, they may be fortified after one or other of the following methods :

1. Suppose the returning angle DCD, which is not obtuse ; take CE and CE, each of 8 or 10 paces, and having drawn the perpendiculars EF of the same length with the lines FG, you will have the exterior circuit of the tenail work DEF G C
- Fig. 30.* of 8 or 10 paces, and having drawn the perpendiculars EF of the same length with the lines FG, you will have the exterior circuit of the tenail work DEF G C
- Fig. 31.* G F E D ; to which may be added an oreillion.
2. Instead of the faces GC, may be drawn G G : Or, from the point C, with the distance CE, describe the arch EGGE, in order to raise a work of strength.
- Fig. 31.* drawn G G : Or, from the point C, with the distance CE, describe the arch EGGE, in order to raise a work of strength.
- Fig. 32.* EGGE, in order to raise a work of strength.

3. If the angle DCD is obtuse, take CG of 25 fathoms, and of the same length draw the perpendiculars CH; so shall *Fig. 33.* the line HH finish the outward circuit of the platform DGHGHD. There may be also added oreillions or epaulements upon the flanks GH.

S E C T. III.

Of Redans.

When a long extended side of a place cannot admit of bastions, by reason it is cut off by a river, or stands upon a steep ascent, it may be fortified with Redans. Suppose, for example, the side of a place AB; divide it into *Fig. 34.* parts of 40 paces or fathoms each in the points C; then draw the perpendiculars CD of 4 or 5 paces, and the oblique lines CD will finish the exterior circuit of the redans ADCDCDB, to which may likewise be added oreillions.

S E C T. IV.

Of Triangles.

There is seldom any occasion of fortifying triangles; however, the best way *Fig. 35.* of doing it, is with bastions, with returning angles.

S E C T.

S E C T. V.

Of Citadels.

1. The Fortifications of Citadels are of less extent than those of the towns themselves, the side of their polygon seldom exceeding 80 geometrical paces.

2. They ought intirely to command the places they are intended to defend; so that the place itself should lie open on the side next the citadel.

S E C T. VI.

Of Sconces.

Sconces are little forts, which serve to defend camps, the lines in a siege, batteries, bridges, &c.

1. Redoubts are square works, one angle of which points towards the enemy; they ought to be about 40 paces in circumference; their rampart 16 foot thick at the bottom; their talus and their height 5 foot; their parapet 5 foot high, and 8 thick; the whole ought to be compos'd of earth well rammed, and fascines; their ditch should be 18 foot wide, and 6 deep.

2. Star Forts are thus made; suppose the radius of the polygon AA to be 110 or 115 paces, divide each side by the perpendiculars BC, then making BD one third of BC, draw the lines AD, and you will

will have the exterior circuit of the little fort BAD, for the rest is much the same as in redoubts.

S E C T. VII.

Of old Places, Castles, Bridges and Camps.

1. As to places fortified after the antient manner, all that can be done, is to add some out-works, as ravelins and horn-works, before the towers and the curtains.

2. Castles are encompassed with a moat and a faulbras of earth well beat, with a kind of bastions, whose magnitude must be determined by that of the castle.

3. Camps are also fortified with ditches, the earth thrown out of which serving to make a kind of rampart, with a parapet circumscribing the camp, and cast up in such a figure as may best defend the several parts of the rampart; redoubts, and ravelins, &c. are also commonly added.

4. Bridges are fortified with ravelins, and crown-works, extending from each side to the river. In one word, all other posts which are to be fortified with expedition, must be secured with such kind of works; care being always taken that every side be well defended.

P R O P.

PROP. VII.

Of Marine Fortifications.

Though Marine Fortifications have nothing peculiar in them, yet it may not be improper to give the following directions with relation to batteries.

I.

In raising batteries to hinder a descent, care should be taken to dispose them in such places where the descent is most easy, as in places where the sea has a clear bottom, the road large and safe, and the shore not steep, the guns upon these batteries should be so levell'd, as to scour the surface of the water, and brought as near as possible to the edge of the shore, that they may fire effectually upon the small boats as they approach; yet ought the battery to be so elevated, as to discover the enemy at a distance.

II.

It is likewise convenient to have batteries to play upon such places where there is good anchorage, and these batteries should be somewhat more elevated, because a cannon, which commands the deck of a vessel, is more terrible to the crew; and besides a bullet which enters a ship from above downwards, puts her into the greatest danger of sinking.

III.

It is also necessary to erect batteries at the entrance of roads; but they must be so made
as

as to discover ships at a great distance ; some of their guns ought to be very much elevated ; but at the same time they should have others raised only 12 or 15 foot above the level of the ground to fire upon ships as they approach the shore.

IV.

It is of the utmost importance to plant batteries which may command the anchorage of roads and ports, to the end that the enemies ships, after having pass'd the fire of the entry, may not remain in safety. Upon extraordinary occasions, they sometime sink ships in the middle of roads, and raise several batteries at proper distances. The same thing may be done at the entry of roads.

V.

It is very necessary that these sorts of batteries should be defended by some works against attacks, and should, if possible, be under the fire of the place, at least they ought not to be too far advanced.

REMARK.

I say nothing of the manner of besieging or defending a place, or of the drawing up an army, these being things which depend more on experience and good sense, than on mathematics.

PROP. VIII.

To trace out Fortifications upon the Ground.

I. Having exactly considered all the parts of the plan, mark the point where you ought to begin, by there fixing a piquet.

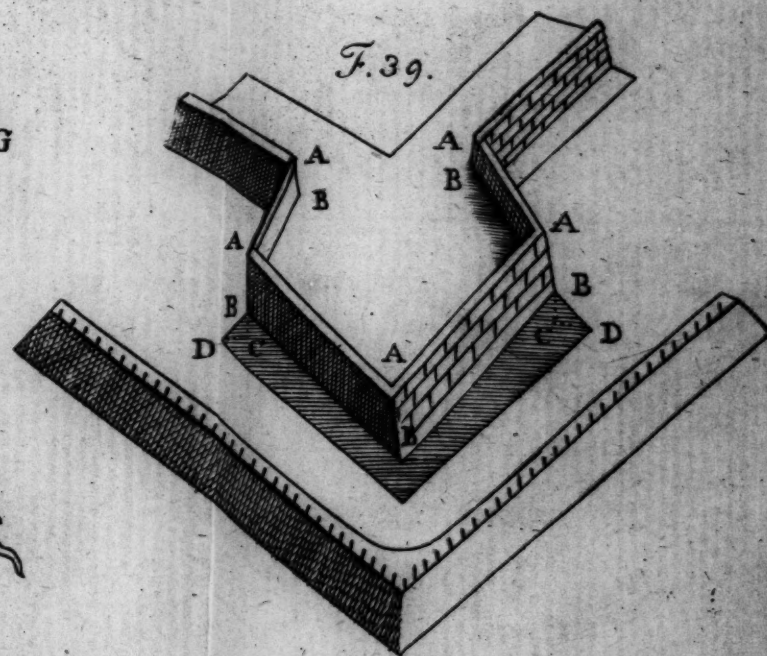
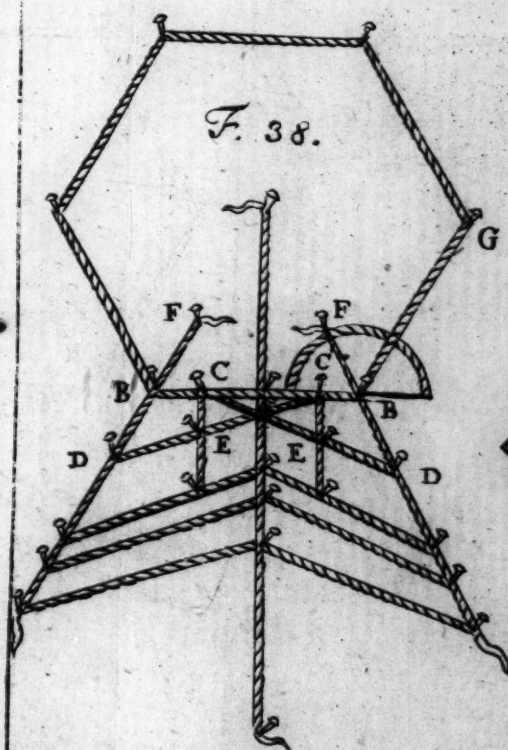
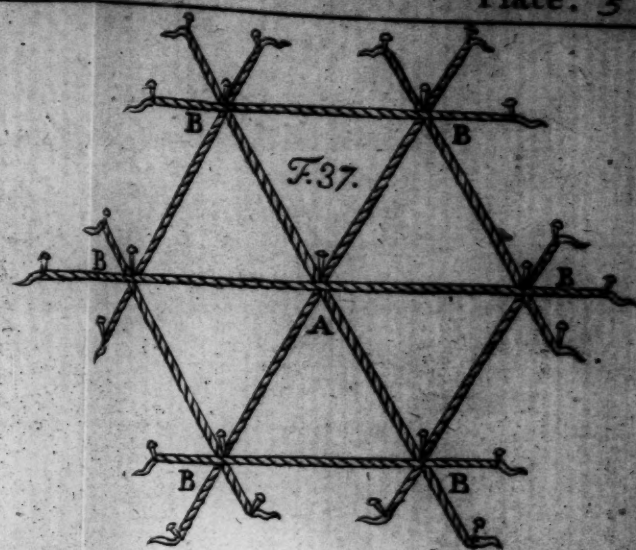
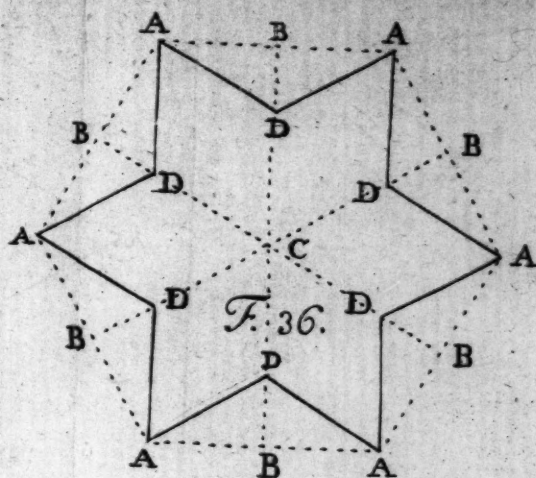
2. If

2. If the determined point be the center of the polygon A, extend the cord AB to the length prescrib'd in the plan; give also to the sides BB their proper length, and that will determine the angles, and form the polygon.

Fig. 37. 3. If the determin'd point B is the angle of the polygon, from the point B extend the cord BF, so that the line BF may tend directly to the center of the polygon, then extending the cord BB to the length required for the side of the polygon, by the help of your instrument, set off the angle FBB as the plan directs; in the same manner must be laid down the side BG of the polygon, according to its requir'd length, and setting off the angle FBG as before, &c. If in proceeding thus with each side of the polygon, the last closes the figure, the operation is right; but if it does not close, you may easily correct the deficiency.

4. The polygon being thus traced out, set off the demigorges BC, and the capital lines BD, and having drawn the perpendicular CE for the flanks, take notice if the points DEC are in the same line, which will serve to correct any error that may have happened; thus will the bastions CEDEC be exactly traced out.

5. Pursuing the same method, you will easily accomplish the whole plan, still serving yourself with the principal lines and perpendiculars drawn by the middle of the curtain.



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P R O P. IX.

To put your Plan in Perspective Orthographically.

1. Having laid down your plan with all the lines which ought to mark the thicknes, draw from each angle A, the vertical lines AB *Fig. 39.* of such length as each point A ought to appear raised above the level of the ground, then drawing the lines BB, and observing all the lines AB and BB, which ought to appear, that is to say, which are not lost in any thickness, or hid behind any body, so will your plan appear raised.

2. If you would express the talus, take the vertical lines BC of the length of the talus's height, and draw the line CD of the length of the talus, and from the most convenient side, and the lines BD shall express the talus.

3. To finish the perspective, we must suppose the light to come from one side, and leaving the faces opposite thereto without shade, give a greater or lesser shadowing to the other faces, according as they more or less are distant from the side whereon the light directly falls.



G U N.

To put your Plan in Perspective Orthographically.

1. Having laid down your plan with all the lines which ought to mark the thickness, draw from each angle A, the vertical lines AB, AC, &c. of such length as each point A ought to appear raised above the level of the ground, then drawing the lines BB, and observing all the lines AB and BB, which ought to appear, that is to say, which are not lost in any thickness, or hidden, and any body, so will your plan appear raised.

2. If you would express the talus, take the vertical line BC of the length of the talus's height, and draw the line CD of the length of the talus, and from the most convenient place, and the lines BD, shall express the talus.

3. To finish the perspective, we must suppose the light to come from one side, and leaving the faces opposite thereto without shade, give a greater or lesser shadowing to the other faces, according as they more or less are distant from the side whence the light directly falls.



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G U N N E R Y.

 T is not the design of this tract to offer every thing necessary to be known concerning Artillery; but only such things in that Art, as have a more particular dependence on the Mathematics.

D E F I N I T I O N S.

1. Suppose a cannon and its profile to be AB, its cavity or chase will be BL, its mouth or muzzle B, its breech or coyle A, its platform E, its vent or touch-hole L, its first re-inforcement I, its second re-inforcement F, its trunnions G, its dolphins H, its vacant cylinder GB, its astragal D, its neck C, its face CB, Fig. 1.

2. The calibre or bore of a cannon is the bigness of its mouth with respect to the ball or bullet it may receive.

3. Cannons are distinguished, with regard to their length and bore, into three kinds, *viz* a Cannon, a Culverine, and a Bastard Cannon. The

The cannon has a length in a certain proportion to its bore; the culverine has the same length, but its bore is only half the diameter of the bore of the cannon; the bastard cannon has also the same length, but the diameter of its bore is a mean between that of a cannon and a culverine.

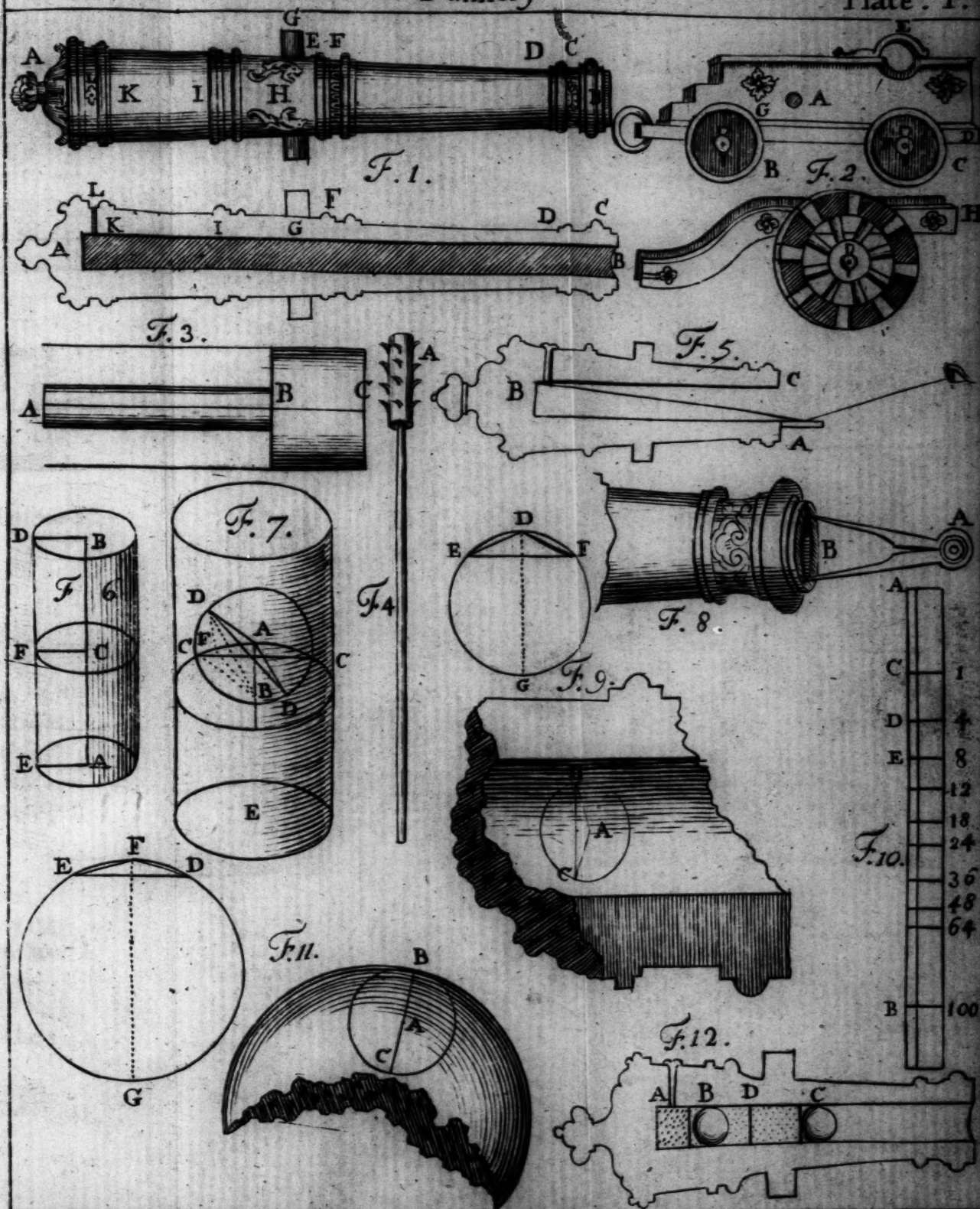
4. The faulcon and faulconet, are small cannons, the first carrying balls but of 10 pounds, the other but of 5.

Note, For sea-service, they use cannons of different bores, that is, carrying balls of different weights, viz. 4, 6, 8, 12, 18, 24, and 36, pounders, because culverines would be too troublesome upon the decks of ships. *

5. Cannons are again distinguished, with respect to the richness or thickness of their metal, into three kinds; viz. the *Double fortified*, the *Legitimate*, and the *Lessened*: The double fortified have above nine times the diameter of its mouth in the outward circumference of its breech; the legitimate has precisely nine times, and the lessened less than nine times.

6. The carriage of a cannon serves to support it; to turn it to the right and left, and to raise or depress it. The carriages used in ships are very fit for all these purposes, as Fig. A, of which the wheels are seldom

* The Cannon used by the *English* in their sea-service are of the following calibres, that is, carry balls of the following weights, viz. $1\frac{1}{2}$, 3, 6, 9, 12, 18, and 24 pounds.



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seldom more than 8 inches radius ; B represents the hind wheels, C the fore wheels, D the bottom of the carriage, A the sides or brackets, E the rests for the trunnions, F the axletrees of the wheels, G the holes for the lashing-fast or securing the piece. In the land-service, they are obliged to mount their cannon upon carriages, better adapted for drawing them, having wheels of two or three foot radius, as Fig. H.

S U P P O S I T I O N S.

The general laws of motion are here supposed, the chief of which are laid down in the beginning of the tract relating to the working of a ship, and some others are here annexed.

1. A body in motion cannot communicate more of motion than is in itself.

2. There are but two ways that motion can be destroy'd ; 1. When the moving body meets with an obstacle it cannot overcome. 2. When it receives a new movement contrary to its former direction. When motion is destroyed by an obstacle it cannot overcome, it is intirely destroyed at once ; but when it is overcome by a contrary movement, it is only destroyed in proportion to the quantity of such contrary motion.

3. When one body in motion meets another, it communicates to it, if it can, a force necessary to put it in motion with an equal velocity with itself.

4. When one body in motion meets another body, it receives from it a contrary movement equal to that it communicates.

5. When a moving body has two contrary and equal motions, if it is intirely at liberty to move itself by their directions, the two several motions will destroy each other, and the body will remain immoveable; but if the body cannot obey the direction of the one, that motion will be destroyed, and the other will remain intire.

6. If a moving body has two contrary and equal motions, and being at liberty to follow the direction of the one in its full velocity, is not at liberty to move according to the direction of the other, but with a part of its velocity, this second motion will destroy a part of the other, in proportion to its own velocity, and will itself be intirely destroyed.

Note, The foregoing laws of motion demonstrate themselves by continual experience.

PROP. I.

The Proportions of the several Parts of a Cannon.

The proportion of the parts of a Cannon depend upon so many circumstances, that experience serves much better to determine them, than any stated rules: So that I shall content myself with giving the following tables, wherein I have inserted only the proportions of such pieces, whose calibres are common both to the *French* and *English* usage.

P R O

P R O P O R T I O N S of Brass Cannons.											
Calibre	Weight	Diameter of the outward Circuit.				Diameter of the Bullet.	Length from the Cascabel, to				Quantity of the Charge, for
		at the mouth	I. L.	I. L.	I. L.		I. Reinf.	2. Reinf.	3. Reinf.	the muzle.	
lb.	lb.	I. L.	I. L.	I. L.	I. L.	I. L.	F. I.	F. I.	F. I.	I. F.	lb.
6	1600	3 6 10	7 9 11 8	10	6 2 3	5 2 4 3	4 3 6 6	8 7 0	6	4 1 2	3 1 2
12	3000	4 6 13	5 12 5 11 2	7 7 4	4 3 3	0 4 1 4	3 8 2 8	6	9 3 4	7 1 3	5 3 3
18	4000	5 1 15	4 14 4 13 3	8 8 4	4 1 1 3	2 4 5 4	8 8 7 9	0	12 1 3	9 1 4	7 1 4
24	5000	5 8 16	11 15 5 14 1	9 5 5	5 3 4 4	10 4 11 9	0 9 6	12 9 1 3	16	12	9 1 3

N. B. I have included in these Tables only such Pieces of Cannon, whose Calibres are common both to the *French* and *English* usage.

P R O P O R T I O N S of Iron Guns.					
Calibre.	Diameter of the Chafe.		Diameter of the Breech.		Whole Length.
	Inches.	Lines.	Feet.	Inch. Lin.	Feet. Inches.
Pounds					
6	3	6	1	0 7	7 0
12	4	5	1	3 5	8 6
18	5	1	1	5 9	9 0

N. B. I have omitted the Table of the Proportions of Sea Carriages, since with us they are adjusted by the Dimensions of the Ports.

P R O P. II.

Of the Cavity or Chase of a Cannon.

The Chase of a Cannon ought to be exactly in the middle, it ought to be equal throughout, and without crevices or honey-combs.

1. Whether the chase be perfectly in the middle, may be tried by a cylinder of wood ABC, whose part AB is equal to the cavity of the cannon, and the part BC equal to its outward circumference. *Fig. 3.* Let the cylinder AB be put into the cannon, and applying a rule all round upon the lines BC parallel to the axis of the cylinder, you will see if it exactly agrees with the outside of the cannon.

2. Whether the chase be equal throughout may be known, by applying rulers to its inward sides; for if the rulers are parallel, 'tis a sign the inward sides of the cavity are so likewise, and consequently that it is equal throughout. This equality of the cavity does not exclude the chamber, which may be made at the bottom of the cannon, as will be mentioned hereafter.

3. To know if a cannon has crevices or honey-combs, after having fired it twice or three times with double charges, they use a sort of rammer armed with flexible points, which closing towards the rammer in thrusting it into the cannon, extend themselves, and entering into the crevices, remain there upon a forcible drawing it back. Another way of trying, is by applying a looking-glass A, *Fig. 5.* to the mouth of the cannon, which

will represent very plainly the opposite side BC, and successively all the rest, by changing the position of the cannon, and consequently the crevices, if there be any, will appear.

PROP. III.

Of the Outside of a Cannon.

With respect to the outside of a Cannon, regard must be had, 1. To the force of the powder and ball. 2. To the parallelism of the sight and the chase. 3. To its beauty and conveniency of movement.

1. A cannon must be thickest towards the breech, because the force of the powder is there greatest, nevertheless its mouth must be also strengthened with rings, lest the bullet, in going out, should split the edges, not being of the same thickness of metal.

2. That the greater thickness of metal at the breech than at the mouth of the cannon, should not hinder the sight or aim, by preventing its parallelism with the chase or cavity, a button or knob should be rais'd upon the upper part of the muzzle.

3. The other parts of the outside of a cannon serve either to its ornament, as the carving, the devices, and inscriptions; or to its movement, as the dolphins and the trunnions.

LEMMA I.

Suppose the rectangular cylinder AB, whose bases AB are circles; I say that all the surfaces of the cylinder parallel

Fig. 6.

parallel to the base A, as C, will also be circles equal to the same base A. Imagine A B the axis of the cylinder, and the rectangle parallelogram ABDE, in turning about the axis AB, to have formed the cylinder.

Demonst. Since the surfaces A and C are parallel, the sections CF, AE, will be also parallel (by 16. 11. of *Eucl.*) and the figure ACFE will be a parallelogram, whose sides AE, CF, will be equal; in the same manner it may be shewn, that the other radii of the circle A are equal to the radii of the circle C, which answer to them, and consequently the circles A and C are equal.

LEMMA II.

In the rectangular cylinder E, suppose a circle parallel to the base E, *Fig. 7.* having for its center B; if from the point A of the circumference of the said circle B, be described a kind of a circle upon the concave surface of the cylinder, it will cut the circumference of the circle B in the point C, and the angle BAC, which the radius AC makes with the radius AB, will be less than any other angle, as BAD, which another radius AD less than AC, would form with the same radius AB. Suppose the line DF parallel to the axis of the cylinder, and perpendicular upon the plane of the circle B, then draw BF, BC, BD.

Demonst. In the triangle BFD, the line BD opposite to the right angle BFD, will be greater (by 19. 1. *Eucl.*) than BF, or BC; and since in the triangles BAC, BAD, the sides BA, AD

are equal to the sides BA , AC , the base BD , which is greater than BC (by 18. 1. *Eucl.*) will make the angle BAD greater than BAC .

LEMMA III.

Supposing, as in the foregoing, a plane to pass through the point D , and the line AB ; the line DD will be greater than CC .

Demonst. In the triangles DAD , CAC , which have the sides AD , AC equal, the angle DAD which is greater than CAC (by the prec.) will make the line DD greater than CC (by 19. 1. *Eucl.*)

COROLLARY.

If the line CC is the least of those, which passing through the line AB , are determined by the circumference of the circle A , the triangle CAC is inscribed in a circle equal to the base E of the rectangular cylinder.

PROP. IV.

Of the Calibre, or Bore of a Cannon.

1. The calibre or bore of a cannon is known by the diameter of its mouth, the cube of which, in comparison with the diameter of a ball of one pound, gives the calibre of the cannon. Thus, when the diameter of the mouth of a cannon is double to that of one pound, the cannon will be an eight pounder; if triple, a 27 pounder; if quadruple, a 64 pounder, &c.

2. To

2. To find the calibre of the cannon B, open the compasses A as wide as they will stand in the mouth of the cannon B. *Fig. 8.*

3. But if it be required to find the calibre from the part A of a cannon; from some point A, describe a kind of a circle upon the concave surface A, and taking the shortest diameter BC, by extending the compasses crossways, as in the figure; then upon the line EF, equal to BC, form a triangle with sides DE, DF, equal to the radii AB, AC; this triangle being inscribed within a circle EDFG, will give DG the diameter of a circle equal to the mouth of the cannon, of which A is a piece, (by *Lem. 3.*) *Fig. 9.*

4. It would be convenient to have a rule, as AB, for measuring the calibre of cannon, of which the part AC being the diameter of a cannon of one pound, AD will be the diameter of one of 4, and AE of one of 8 pounds; so that by applying the diameter of a cannon to the rule, you will at once see its calibre. *Fig. 10.*

Note, The rule for measuring the calibre, is divided in the same manner as the rule for measuring solids on the Sector, or Compass of proportion. (*vide Appendix.*)

PROP. V.

Of the Bullet.

A Bullet is commonly made of a solid globe of iron.

1. It is made of a spherical figure, as the most proper to receive, keep, and communicate motion.

2. It is made of iron, as being a metal of sufficient hardness and weight, and of which there is a sufficient plenty.

3. It is made solid, because a hollow bullet would be liable to the same resistance from the air, and the body it strikes; but would want the force which solidity gives it.

PROP. VI.

Of the Calibre of the Bullet.

1. The calibre of a bullet is in the same manner known by its diameter taken between the points of a pair of Callipers, or curve-legged compasses, the cube of which diameter, with respect to one of the weight of one pound, gives its calibre, and the proportion is the same in bullets as in cannons; only it ought to be observed, that the diameter of a bullet ought to be less by two lines, than the diameter of a cannon of the same calibre, that so it may enter easily.

.2 On

2. On the other hand, we know the diameter of a bullet by its weight, or calibre ; for the cube root of its weight being divided by the cube root of the weight of a bullet of the same metal, having unity for its diameter, gives the diameter required.

3. This shews a new way of dividing a rule for taking the calibre. For let AB represent a rule, of which the part AC is *Fig. 10.* the diameter of a bullet of one pound weight ; let the whole rule be divided into 1000 equal parts, of which let AC be 10, then adding 3 cyphers to the weight of each bullet, extract the cube root, and you find the number of parts answering to the diameter of each bullet, almost to a thousandth part. For example, if you would mark upon this rule for the calibre, that of a bullet of 24 pounds, adding 3 cyphers to 24, which makes 24000, the cube root thereof, viz. 29, gives the diameter of a bullet of 24 pounds. After which manner was composed the following table.

Weights of Bullets in Pounds.

1	2	4	6	8	12	18	24	36	48	64	100
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
10	13	16	18	20	23	26	29	33	36	40	46

4. To find the diameter of a bullet, of which we have only the convex part A ; on the point A, as a center, describe a circle upon the convex surface A, and having taken the diameter BC with a pair of callipers, then *Fig. 11.* upon the line ED equal BC, make a triangle,

K 5

triangle with the sides FE , FD equal AB , AC , &c this triangle in the circle $EFDG$, the diameter of which FG , will be the diameter of the bullet.

Demonst. The plane of the triangle ABC passing through the pole A , and through the diameter BC of a circle, will pass through the center of a globe (by Prop. 1. *Sph. Trig.*) therefore the triangle ABC will be inscribed in a circle, which will pass through the center of the bullet, and consequently will have for its diameter that of the bullet.

PROP. VII.

Of the Cause of the Motion of the Bullet in a Cannon.

Let AB represent a Cannon, wherein *Fig. 12.* the powder takes up the space A , and the bullet the space B ; when the powder takes fire, it will rarify the air and fill a larger space, as AC , and thrust the bullet from B to C ; from whence may be observed,

1. That the swiftness of the motion communicated to the bullet by the powder, ought to be measured by the line AC , the length of the space gained by the powder in firing; because in the time of the powder's firing, the bullet is forced with the velocity AC .

2. Whether the powder is inflamed successively, or all at once, the bullet receives an equal velocity, since it all takes fire before the bullet goes out of the cannon. For if at the first instant only half the powder should take fire, instead of *Fig. 12.* gaining the space AC , it will only gain AD , and communicate to the bullet only

only the velocity AD; but afterward as the rest takes fire, it will communicate a second velocity equal to DC.

Or it might have been thus.

Let AE represent a Cannon, wherein the powder takes up the space A, and B is the bullet; the instant of the powder taking fire, the flame by expanding the air will with great force press the bullet forward, and fill a larger space, as AD, AC, or even AE the whole bore; from whence may be observed,

1. That the swiftness of the motion given to the bullet is by the instantaneous expansion of the air in the fired powder, and that such swiftness continually increases, whilst the bullet is moving the whole length from B to E, if the expansion of the flame is of force sufficient to press out the bullet, if placed at rest in E.

2. That the height of the charge of powder should be proportioned to the diameter of the bore and length of the cannon, as a heavy bullet will require a greater force to give it a velocity equal to a lighter in proportion to their weight, and if the length of the cannon is increased, the expansion of the flame must be increased, so as to press the bullet for the whole length, or else the length will diminish the carriage of the bullet.

3. That the increase of the weight of the bullet helps to the continuance of its velocity, as it has thereby the more force to overcome the obstacles it meets with.

4. That

4. That the increase of the weight of the bullet diminishes the expansion of the flame of the powder; for though the bullet has not any contrary motion, yet as it must receive its whole motion from such expansion, it will, in resistance thereto, communicate a contrary motion to that it receives, and such contrary motion increases in proportion to the weight of the bullet.

C O R O L L A R I E S.

1. If two equal bullets, A and B, are
Fig. 13. thrown out of equal cannons, their velocities will be in the same proportions as the quantity of powder in the one and the other cannon, since the spaces AC and BD will be in the same proportion as the spaces A and B.

2. If the bullets A and B, of different
Fig. 14. calibres, be thrown out with charges proportional to the cubes of their depths, their continued velocities will be as the cube roots of their calibres, since the lines AC and BD increase in the same proportion as the cube roots of the quantities of the charges.

3. It will be easy to determine the proportion of velocity betwixt bullets of different calibres, whose several charges are determined. For 1. If the charges are proportioned as above, their continued velocities will be as their diameters. 2. In diminishing or augmenting the charges, their velocities are diminished or augmented in the same proportion.

Mr. Robins, in his new principles of Gunnery (hereafter mention'd) proves that all the powder of the

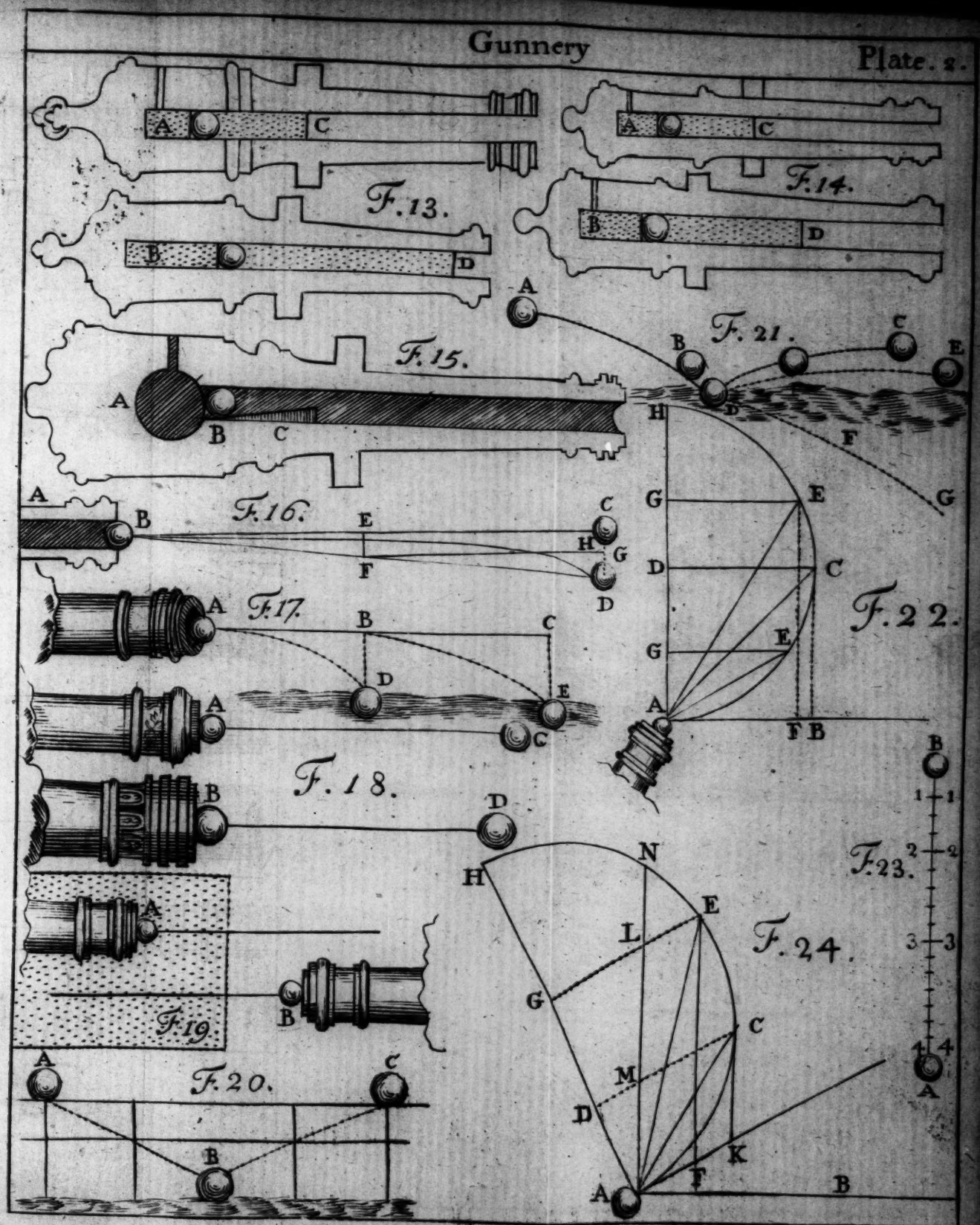
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the charge is fired, before the bullet is sensibly moved from its place; and after determining the force of our government gun-powder, gives rules (that have relation to the length and calibre of the cannon and bullet) for finding the velocities given by different charges to all kinds of bullets, and the charges that will give the greatest velocity to the different kinds of cannon and bullets.

P R O P. VIII.

Of the Chamber, where the Powder is plac'd in a Cannon.

1. The chamber A serves to inflame *Fig. 15.* a greater part of the powder; for the powder A taking fire in the middle, is forced against the bounds of the chamber, from whence it is reverberated against the fire.

2. Nevertheless, chambers are seldom made in cannons, because they would thereby be rendered more difficult to be charged and discharged, and after firing oftentimes, some of the fire would remain, which would be dangerous to the Cannoniers.

P R O P. IX.

Of the length of a Cannon.

1. It is certain, that the length of a cannon diminishes the motion of the ball, by its rubbing against the sides in passing.

2. The

2. The length of a cannon serves to augment the motion of the bullet, because it gives time to the powder to take fire in a greater quantity before the ball gets out.

3. Experience only can determine the length of a cannon, because its effect depends upon certain circumstances, which cannot be exactly known; as the quality of the powder, the resistance the bullet finds in rubbing, &c.

P R O P. X.

Of the charging a Cannon.

1. Tho' experience must regulate the quantity of powder, yet in general it may be observed, that in proving a cannon, the charge may be much greater than in battle, or when the cannon grows hot.

2. The bullet must enter easily, lest it damage the metal in going out, or, it may be, burst the cannon. Care should therefore be likewise taken, that there be no inequalities in the ball, at least that it will enter easily every way.

3. It would be easy to burst a cannon, if after having put in the bullet B, the wedge C be insinuated of a sufficient hardness to resist the force of the bullet, for then the bullet cannot pass without bursting the cannon.

P R O P.

P R O P. XI.

Of a Cannon's heating.

1. When a cannon grows hot, it recoils with greater violence upon the carriage, because the powder takes fire quicker, and in a greater quantity.

2. Although the powder strikes equally against both the upper and lower parts of the cannon, yet notwithstanding it will recoil; since it is not at liberty to move downwards; for the lower part being forc'd against the carriage, receives from that resistance a contrary motion (by *Sup. 4.*)

3. The heat of the cannon commonly increases the velocity of the bullet; for that it causes a greater quantity of powder to take fire before the bullet goes out of the cannon; yet it may happen to shorten the carriage of the ball, if the cannon recoiling, falls again before the bullet gets out; for that motion of the fall of the cannon being communicated to the bullet, very much diminishes the length of its carriage, as shall be shewn farther on.

P R O P. XII.

Of the horizontal Carriage of a Cannon.

Let the cannon AB be disposed horizontally, the bullet B will not *Fig. 16.* keep the horizontal line ABC, but a curve line, which will carry it to the point D below the point C.

Demonst.

Demonst. The horizontal motion impress'd on the bullet by the powder, not being contrary to the motion of its own gravity, will not destroy it; therefore the bullet will be carried downwards, being moved by a compounded motion, partly horizontal, and partly vertical, which will consequently carry it to the point D below the point C.

C O R O L L A R Y.

From whence it follows, that when a cannon seems to carry point-blank, or to raise its stroke, the line of aim is not parallel to the cavity of the cannon, for then its carriage would be too low.

R E M A R K S.

I. It is not necessary to examine the nature of the line described by the bullet in its passage, it being demonstrable, that it would be parabolical, if the resistance of the air, or the directions of its own gravity, did not affect it. It is sufficient to know, that the line BD, the passage of the bullet, is not the right line BFD. Let the right line BFD be therefore equally divided in the point F, and draw FE parallel to CD, and FG parallel to BC.

Demonst. The line EF being equal to the line GC, the bullet will take up the same time in passing from the point B to the line EF, by reason of its horizontal motion, as in passing from the line EF to the line CD; therefore when it comes to the line EF, it will have descended

scended but the fourth part of the line CD (by *Prop. 10. Mechan.*) and not its half part EF. Therefore the passage of the bullet will be above the point F.

2. The greater the velocity of the bullet is, the less will be its descent at the same distance; because it takes up a less time in its passage, and the quantity of time determines the space or quantity of its descent. So that the bullet which was carried from B to D, would have been carried from B to H, if it had been thrown by the cannon with a double velocity; because it would then have taken up but half the time in passing from the point B to the line CD.

3. If the curve line of the bullet's passage did not bring it to the ground, it would not stop till the resistance of the air intirely destroyed its horizontal motion, and its progress would consequently become immense.

4. Horizontal carriages are to one another, according to their several velocities; if no regard be had to the resistance of the air.

Demonst. If in the same time that one bullet passes, by its horizontal motion, from the point A to the line CE, another *Fig. 17.* bullet is only carried from the point A to the line BD; if further, the first bullet in its passage from A towards C, falls upon the ground at the point E, the other bullet falls equally, and strikes the earth at the point D; consequently as AC the measure of the velocity of the first bullet, will be the measure of its carriage; so AB the
measure

measure of the velocity of the second bullet, will also be the measure of its carriage.

PROP. XIII.

Of the resistance of the Air.

1. The resistance of the air retards the velocity of the bullet by the particles of the air repelling it.

2. The resistance of the air increaseth in a duplicate ratio of the velocity of the bullet; for one bullet having twice the velocity of another, strikes against a double quantity of air in the same time, and likewise with a double force.

3. The resistance of the air diminisheth the carriage of a bullet in a duplicate ratio of its velocity:

Or it might have been thus,

4. *The resistance of the air to the carriage of different bullets, projected with equal velocities, is in proportion to the squares of their diameters, but not to their mass or quantities of matter; these being in proportion to the cubes of their diameters.*

Demonst.

Demonst. The bullets A and B having the same velocity, will find the resistance of the air in proportion to the areas of the space each passes through ; and therefore not in proportion to their masses or quantity of matter. Fig. 18.

COROLLARY.

Though the carriage of one cannon be given with all its circumstances, we cannot from thence determine the carriage of another exactly, without regard to the resistance of the air, which cannot certainly be known.

5. The denser the air, the greater the resistance, and the carriage of the bullet will be diminished in proportion to such density, since the bullet must consequently strike against a greater number of particles.

6. If two equal bullets pass through different mediums of air with different velocities, the resistances they meet with will not be equal. But supposing the velocities reciprocally proportional to the densities, the bullet which passes through the space of denser air, will find less resistance in the same proportion as their velocities.

Demonst. If one of the bullets pass through air twice as thick as the other, and the other with double velocity to the first, they will then both be struck with an equal number of particles of

of air ; but the second will be struck with a double force.

COROLLARY.

If the bullets A and B, equal in every respect, are thrown with equal force, and the bullet A passes through half its course in a thick air, and the rest through a more refin'd air ; if on the contrary, the bullet B pass at first through a finer air, and continues the rest of its course through a thick air ; these two bullets will find equal resistance, and will be equal in their carriage, supposing the air A twice as thick as the air B.

Demonst. The bullet A being come to the middle of the thick air, will be so much overcome by the resistance it meets with, and will have lost so much of its velocity as the bullet B, at the end of the thin air ; since it will have struck against an equal number of particles, and with an equal force. Again, they will find an equal resistance in the remainder of their course, the bullet A in passing through the remaining half of the thick air, and the bullet B in passing through the first half of the thick air. Further, the bullet A will find the same resistance in passing through the thin air B, as the bullet B in passing through the remainder of the thick air A.

The ingenious Mr. Benj. Robins F. R. S. has lately published new principles of gunnery, wherein he determines the force of gun-powder, and investigates the difference of the resisting power of the air to swift and slow motions ; and shews by several experiments,

ments, that its resistance to swift motions is vastly greater than according to the foregoing doctrine, which he says, holds true only in slow motions; and that if the bullet has the velocity given to it equal to the motion of 1700 feet in one second of time (which he found by several very curious experiments made by the bullet impinging into a pendulum, to be nearly the greatest velocity given to a musket or a cannon ball) the resistance of the air is greater than in the doctrine of slow motions nearly in the ratio of 3 to 1; and that if the flight of a musket-ball of $\frac{3}{4}$ of an inch diameter, fired with half its weight of powder, from a piece 45 inches long, with a velocity equal to 1700 feet in 1", was in the curve of a parabola, its horizontal range at 45° elevation would be about 17 miles; whereas all agree that this range is really short of half a mile; therefore the track described by such a bullet greatly differs from a parabola, as it flies not the $\frac{1}{34}$ part of what it ought to do, if moving in that curve; and this contraction will not be wonder'd at, when it shall be considered, that the resistance the air gives to this bullet at its first issuing from the piece, is about 120 times the weight of the bullet.

He also proves that a cannon-ball of 24^{lb} fired with 16^{lb} of powder (which is the usual battering charge) acquires the velocity of about 1650 feet in a second, and the air's resistance to this ball, at its first issuing from the piece, is more than 20 times its weight, and that if the flight of this ball was in the curve of a parabola, its greatest horizontal range would be about 16 miles, which is between 5 and 6 times its real quantity, as it will always fall short of 3 miles.

P R O P. XIV.

Of the resistance of Water, and the reboundings of the Bullet therein.

Water makes a greater resistance than air, both because it is a more dense body, and that its parts are more difficult to divide; therefore bullets meet with so great resistance in water, that they sometimes rebound, and make, as it were, repeated leaps upon the surface of the water, as in the figure.

Suppose the bullet A, which is carried against the surface of the water B, along the curve line AB inclined upon the same surface. 1. If the bullet was without weight, and the water solid at the point B, it would be forced back through the line BC, equal in every respect to the line BA, since its motion parallel to the water would remain the same, and the water would communicate to it a motion repelling it, equal to that with which it approached (by *Sup. 4.*)

2. The water not being solid, the air preceding the bullet, will make, as it were, a cavity in it, in such manner, that the bullet instead of striking it at the point B, will not meet with it but at the point D; or it will enter into the water by some curve line as DG, if it has force enough to divide it at the point D, as if it were thrown against the water by a line less inclined.

But

3. But if the bullet is carried with so much velocity, and so obliquely against the water, that it cannot divide it, it then receives a motion repelling it from the water with a force equal to that with which it approached it; and in such case, if the bullet was without weight, it would be repelled by the line DC equal in every respect to the line DBA.

4. The weight of the bullet causes the line DE, through which it is repelled, to be much more inclined than the line DC; so that the bullet rises out of the water at the point F, after having pass'd under the water through the whole space BDF.

LEMMA I.

If from the end A of the diameter AH, be drawn the lines AC, AE, extended *Fig. 22.* to the circumference of the circle, and from the points CE be drawn the lines CD, EG, perpendicular to the diameter; the line AD will be to the line AG, as the square of the line AC to the square of the line AE.

Demonst. The square of AC is equal to the squares of AD and DC (by 47. 1. *Eucl.*) and since the square of DC is equal to the rectangle ADH (by 32. 3. *Eucl.*) the square of AC will be equal to the square of AD, and the rectangle ADH; or it will be equal to the rectangle HAD: In the same manner the square of AE will be equal to the rectangle HAG: But as the rectangle HAD: AD: is to the rectangle HAG:: AD: AG (by

(by 1. 6. *Eucl.*) therefore as the square of AC : square AD :: AD : AG.

COROLLARY.

Since (by *Prop. 10. Mechan.*) the descents of heavy bodies are to one another, as the squares of their times; if the times be measured by the lines AC, AE; the descents must be measured by the lines AD, AG. Thus, supposing that in the time measured by AC, a heavy body in its descent would pass through the line AD; then would it run through the line AG in the time measured by AE.

LEMMA II.

If the body A receives at the point A
Fig. 23. a motion upwards, which motion being progressively destroyed by gravity, ceases at the point B; it will be just the same time in its descent from B to A, as it took up in its ascent from A to B; because its motion of gravity will begin and increase by the same degrees, as its former motion decreased and ended.

COROLLARY.

Suppose the line AC elevated above
Fig. 22. the horizontal line AB, and the bullet A thrown to the same height, with a motion which would carry it to the point C in a minute of time, if the ball was without weight; but that by reason of its gravity it descends again to the point B in the vertical line CB; we may thence

thence deduce the following consequences: 1. The ball will arrive at the point B in the same time it would be in coming to the point C; since its gravity makes no alteration in its horizontal motion, which carries it from the vertical line AH, to the vertical line BC. 2. The ball will take up one half of the said time in its ascent, and the other half in its fall. (by *Lem. 2.*) 3. The ball will ascend but the fourth part of the line BC, or half that height it would have ascended, in half the said minute of time (by *Prop. 10. Mechan.*) if the motion causing its ascent had continued always equal, and its gravity had not insensibly overcome it.

P R O P. XV.

Suppose from the mouth of a cannon A, whose elevation is AC, a ball being thrown should fall upon the point B of the horizontal line AB. Draw AH and BC perpendiculars upon the line AB, and from some point as D, describe the circle ACH. I say, that if the cannon be set to any other elevation as AE, every other particular remaining the same, the line EF perpendicular to AB, will determine the point F, where the ball will fall.

Demonst. Since the ball, at the elevation AC, falls upon the point B, it must needs be that during half the time it would take up in passing through the line AC, if its gravity did not affect it, its weight will cause it to descend the fourth part of the line BC (by *Cor. prec.*) But if during the time measured by half the line AC, the ball

VOL. I.

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descends

descends one fourth part of the line BC or DA ; then during the time measured by half the line AE, it will also descend one fourth part of the line AG or EF ; (by *Lem. 1.*) therefore the point F will be the place where the ball will fall at the elevation AE.

P R O P. XVI.

Suppose the line AK raised above
Fig. 24. the horizon AB, and the ball A having the elevation AC, to fall upon the point K : Draw AN perpendicular to AB, and KC parallel to AN. Again, let AH be drawn perpendicular to AK, and from some point of the line AH, describe a circle ACH. I say, that if to the ball A be given any other elevation as AE, the line EF parallel to the line AN, will determine the point F, where the ball will fall upon the line AK. Draw CD and EG, perpendiculars upon AH, which will cut AN in the points ML.

L. n^o. The square of AC is to the square of AE ; as AD to AG (by *Lem. 1.*) or as AM to AL (by 2. 6. *Eucl.*) or as CK to EF. Therefore (by the prec.) if the ball A having the elevation AC fall upon the point K, it will fall upon the point F, if it obtains the elevation AE.

R E M A R K.

If the carriage and elevation of a cannon be given, together with the plane upon which the ball is to fall ; we have all others upon every kind of
of

of plane; because the force of a cannon may always be measured by the vertical line AN, which ought to be the same whether the plane be horizontal or not, since the plane has no effect upon the vertical carriage, by which the ball is returned towards the point A.

C O R O L L A R Y.

If the line KC be a tangent to the circle, the line AK is the greatest carriage, and BAC the angle of elevation equal to 45 degrees, with half the angle of BAK; for the angle CAK is equal to half the angle NAK, which is 45 degrees, less half HAN, or BAK.

From whence is deduced this general rule :

To have the greatest carriage upon a plane elevated above the horizon, to the elevation of 45 degrees, must be added half the elevation of the plane.

P R O P. XVII.

The same reasoning holds good, when the line AK lies inclined below *Fig. 25.* the horizontal line AB, after having describ'd the semi-circle ACH, touching the line AK.

C O R O L L A R Y.

In the same manner, To have the greatest carriage AK, deduct from the elevation of 45 degrees half the angle BAK.

PROP. XVIII.

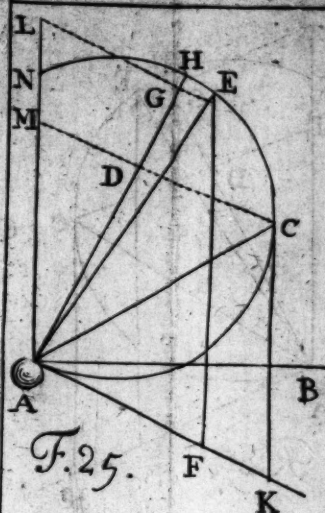
Of the Instruments used to find the elevation of a Cannon.

1. Having a square of wood, or of
Fig. 26. metal, as AB, one of whose sides AC, let be prolong'd at discretion to D. From the point A, describe the quadrant CE, in such manner that AB may be perpendicular to AD; then divide the quadrant EC as usual, beginning from the point E, and let a plummet be hung upon the point A: Lastly, let the whole square AB be filled up with lines parallel to AD, and equidistant from one another.

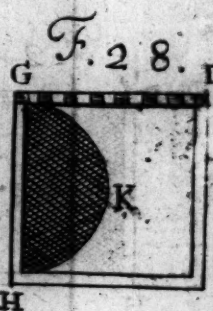
2. To use this instrument, put the
Fig. 27. side CD into the cannon, applying it close to the under side of the cavity or chase, so shall the plumb line AF shew upon the arch EC, the angle FAE, equal to the angle ABF, the elevation of the cannon above the horizontal line BF.

3. Besides the above, there may be
Fig. 28. provided a square of paste-board, or of metal, whose sides GH, GI must be equal to the sides AC, AE of the other instrument; then describing upon GH a semi-circle, which space must be cut away and left open, and divide the side GI into several equal parts.

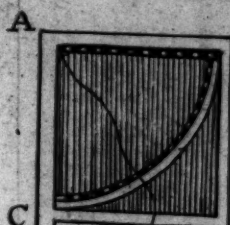
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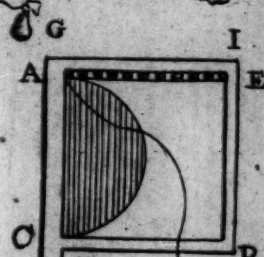
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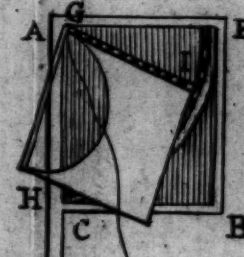
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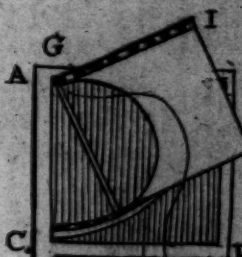
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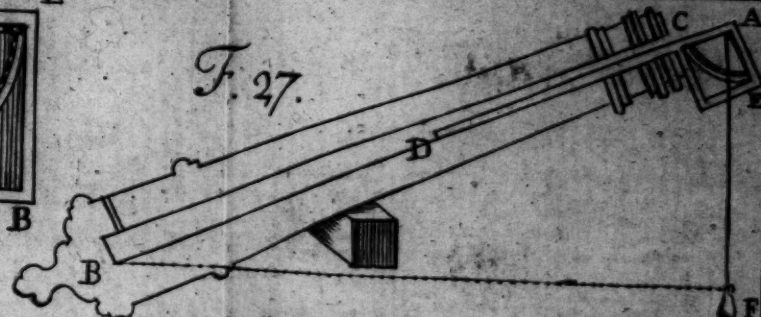
F. 29.



F. 30.



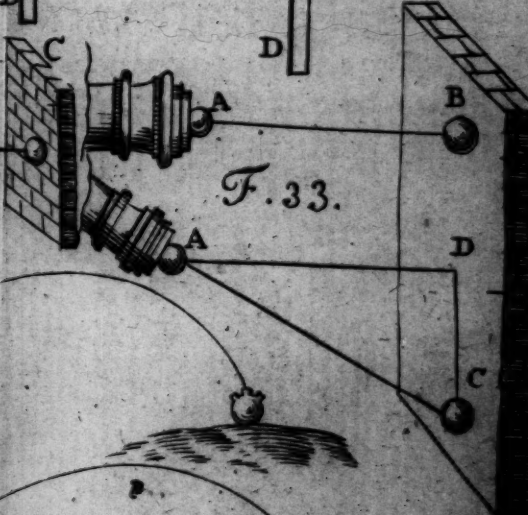
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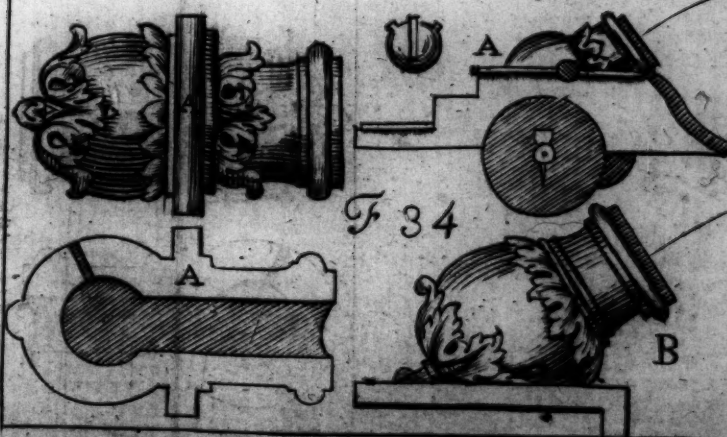
F. 27.



F. 32.



F. 33.



F. 34.

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4. Its use is as follows; when you would determine the horizontal carriage, place the point G of the paste-board upon *Fig. 29.* the point A of the other instrument, and the line GI, upon the side AE; so shall one of the parallels to the line AD, touching the open semi-circle in the point K, shew upon the scale GI, the greatest horizontal carriage at the elevation HK; the other parallels will likewise shew the other carriages upon the scale GI, with their correspondent elevations. Having therefore fix'd upon a certain carriage, put the instrument into the cannon, and raise the piece till the plumb line cuts upon the semi-circle GKH, the correspondent elevation.

5. When we desire to know carriages elevated above the horizontal, we place the side GI of the paste-board under the line *Fig. 30.* AE, so that the angle EGI be the angle of the carriage with the horizontal. If on the other hand, we would know a carriage inclined below the horizon, we *Fig. 31.* elevate the line GI above the line AE, to the same angle. All the rest is done as above.

R E M A R K.

Carriages may be determined, not only by the greatest, as has been above laid down, but even by any other, whose length and elevation is known, since it only depends upon determining the value of the scale GI.

PROP. XIX.

Of the force of the Ball against the Body it strikes.

1. It is plain that a ball striking a body soon after it passes out of the mouth of a cannon (every thing else being equal) will strike it with greater force than after its motion is diminished by the resistance of the air.

2. If the ball A, at its passing out of *Fig. 32.* the mouth of a cannon, immediately meets with the body B, it may happen, that the air forc'd by the flame coming out of the cannon, and compress'd against the body B, may repel the ball with a force that may lessen its motion more than the air it would meet with at a little greater distance; and in that case the ball will strike the body B with less force, than it would strike the body C at a little greater distance, beyond the reach of the flame of the powder.

3. If the ball A is carried against *Fig. 33.* the body B, in the perpendicular line AB, it will communicate its whole motion; but if the ball passes through the oblique line AC, the motion it communicates will be only in the ratio of the line AC to the line AD.

COROLLARY.

From whence it follows, that balls strike with more force against rough stones; than against smooth ones.

PROP.

PROP. XX.

Of Mortars for Bombardments.

Mortars are a sort of cannon shorter and larger than those before described; *Fig. 34.* their use is to throw bombs, and carcasses; however, all that has been said of other cannons may be applied to these, and particularly the rules given for determining their carriages and elevations; what more peculiarly relates to them may be comprehended in the following observations:

1. Mortars ought to be very short, that the bomb may be plac'd in a proper situation, that neither the fuse may be lost in rolling through the cannon, nor the bomb burst in its passage, nor even retarded by rubbing against the sides of the cannon.

2. Mortars commonly have chambers, that the powder may the sooner take fire in a greater quantity, there being no danger that fire should remain in them unperceiv'd.

3. Some mortars, as A, are so contriv'd, as to be rais'd or depress'd upon their carriages, for the better regulating the distance of their carriage; but this sort of mortars are more liable to be dismounted, and more difficult to be fix'd in battery.

4. Mortars which, as B, are cast with their carriage, are generally more esteemed. They are commonly made to the elevation of 45 degrees, because

because that elevation gives the greatest horizontal carriage. Nevertheless, as mortars are often designed to throw bombs into places elevated above the plane they are fix'd in, I would advise the giving them a greater elevation upon their carriages: And if there be occasion to lessen the carriage of such mortars, it may be done by diminishing the quantity of powder in their charging; which seems more natural than lessening the carriage, by giving the mortar an elevation less proper to throw the bomb.

F I N I S.

